On the Global Supply of Basic Research∗

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Abstract

In this paper we study the incentives for basic-research investments by governments in a globalized world. For this purpose, we develop a two-country Schumpeterian growth model in which each country chooses its basic-research investments. We find that a country’s basic-research investments increase with the country’s level of human capital and decline with its own market size. This may explain the large basic-research investments by small open economies. Compared with the optimal investments achievable when countries coordinate their basic-research policies, a single country may over-invest in basic research. However, in the decentralized case the total amount of basic-research investments is always below the socially optimal investment level, which justifies policy coordination in this area.

Keywords: basic research, public goods, economic growth, coordination of governments

JEL: O31, O38

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1 Introduction

Basic-research investments are arguably a core driver of economic growth in industrialized countries. Traditionally, they are a matter of national policy-making. In some areas, however, international cooperation and coordination are playing an increasingly important role. This is most pronounced in the European Union, where large research programs are funded by member states and designed and operated at Union level in Brussels. Moreover, the basic research undertaken at several major institutes such as CERN in Geneva or by other high-technology ventures such as ARIANE are the result of joint efforts and agreements between several countries. Whether international coordination on basic research investment is considered necessary depends both on the way we conceptualize basic research and on the way how investments in one country affect growth and welfare in other countries. There are arguments for and against the coordination of basic research across countries.

- When basic research is viewed as a global public good whose output is freely available and whose consumption is non-rivalrous and non-excludable (Arrow, 1962, Nelson, 1959), the standard “free-rider argument” suggests that uncoordinated investment decisions will entail considerable under-investment.

- Basic research may also be viewed as a regional good with international spillovers. The ideas created by basic research are non-rival goods in the country where these ideas have been generated. As a consequence, basic research may induce and increase prospects of success for regional firms’ innovation efforts.\(^1\) Moreover, firms with successful innovations may be able to increase the rents generated by these innovations through exports or foreign direct investments. The possibility of capturing rents in foreign markets by taking away business from established firms suggests that basic-research investments have negative externalities on other countries, which would cause over-investment.\(^2\)

- When the benefits of basic research are embodied in new products and services,\(^1\) The positive side-effects occur through various channels whose outputs are: supply of trained scientists and problem-solvers, new scientific instrumentation, network for knowledge diffusion, enhancement of problem-solving capacities, start-ups and spin-offs from universities, prototypes of new products and processes (e.g., Salter and Martin, 2001, Brooks, 1994, Moverey and Sampat, 2005, and Gersbach et al., 2009).

\(^{2}\) The negative and positive externalities described in this paragraph are well documented in the literature (Baily and Gersbach 1995, Keller and Yeaple 2003, Alfaro et al. 2006)
and if a country is open to foreign direct investments, this country could benefit from the basic research of other countries. Foreign direct investments by leading-edge firms directly contribute to higher levels of productivity by transferring the best production techniques and products to the host country, thereby raising wages and consumer surplus. These positive externalities suggest that countries tend to under-invest in basic research.

In this paper we develop a framework to study the direction of externalities of basic-research investments and examine whether there is an under- or overprovision of such investments when each country acts on its own. We consider two large countries that select their basic-research investments in each period. Such investments foster the innovation prospects of domestic intermediate firms.\(^3\) Firms that develop leading-edge technologies in one country obtain patents and can enter foreign markets through foreign direct investments to earn monopoly profits. When another country invests more in basic research, a country will experience positive and negative externalities of the kind described above. Moreover, if both countries invest in basic research, this increases the risk that innovation efforts may be duplicated in the world. We study decentralized basic-research investments when governments maximize the consumption of the current generation, and we explore the long-term consequences of such decisions. There we determine the basic-research levels when countries coordinate their decisions. Finally, we study the path of uncoordinated and coordinated basic-research decisions when governments maximize the welfare of all generations.

Our main insights are as follows: First, we show that the countries’ basic-research investments act as strategic substitutes. Further, a country’s basic-research investments will increase with its level of human capital, but decline with its relative population size. The reason for the latter is that a small country can earn large profits from gaining a monopoly position in a larger foreign country without sustaining the corresponding deadweight losses accruing abroad. This result may explain the large basic-research investments made in small open economies such as Korea or Switzerland.

Second, comparing the decentralized basic-research investments with the optimal ones when countries coordinate to maximize aggregate consumption, we find that both countries under-invest in the decentralized equilibrium if they are similar with respect to

\(^3\)Hence, we view basic research as a non-rival good in countries at the technology frontier where new ideas are created. See Jones and Romer (2010) for systematic reasoning on why ideas in applied research should be viewed as partially excludable non-rival goods.
human capital levels and population sizes. Under asymmetry concerning either of these characteristics, one of the countries may over-invest in the decentralized equilibrium relative to the coordination optimum. From the cooperative perspective, however, the aggregate decentralized basic-research investments are too low, even if one of the countries over-invests in basic research.

Third, our robustness discussion reveals that our results do not change qualitatively when we consider a one period or an infinite planning horizon of governments. Of course, investments in basic research increase quantitatively when longer time horizons are considered. In the appendix, we also discuss the implications of different welfare objectives under coordination and different assumptions on the costs of basic research.

The paper is structured as follows: In the next section we relate our paper to the relevant literature and discuss the significance of basic research. We introduce the model in Section 3 and derive the decentralized equilibrium in Section 4. The dynamics of the model are described in Section 5. In Section 6 we compare the decentralized basic-research investments with the optimal ones when countries coordinate to maximize aggregate consumption. Finally, we discuss the robustness of our results with respect to infinite planning horizons in Section 7 and conclude in Section 8. The proofs, as well as some further robustness discussions, are relegated to the appendix.

2 Relation to the Literature and Significance of Basic Research

It is useful to put the significance of basic research in perspective. The empirical pattern of basic research is shown in Table 1.

Two observations are worth emphasizing. First, basic research is mainly undertaken by industrialized countries that are at, or close to, the technological frontier. Some of the emerging countries, such as Korea or Singapore, have considerably stepped up their basic research efforts. Second, large industrial countries such as the U.S. or France spend about 0.5% of their GDP on basic research. By contrast, Switzerland invests a substantially higher share of about 0.8%. Finally, the OECD data (OECD 2009) also show that with the exception of Japan, Korea, and Singapore, the vast majority of basic research is performed in the government/higher education sector.

The theme and the model of our paper are influenced by two lines of research. First,
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** 1993 data
§ 1995 data
* 1996 data
† 2006 data
‡ 2008 data

there is a large body of literature on the importance of basic research in the innovation process and on the strength of international spillovers. Some major articles have already been referred to. Second, our paper is related to the theoretical literature that incorporates basic research into R&D-driven growth models (e.g. Arnold 1997, Cozzi and Galli 2011a, 2009, 2011b, Gersbach et al. 2009). Most of these contributions focus on the optimal level of basic research in closed economies. There are two papers that also investigate open economies. In a two-country model, Park (1998) analyzes how cross-country knowledge spillovers affect the optimal level of public basic research, while the degree of openness determines how large the spillovers are. However, regardless of the degree of openness, the knowledge spillovers come free of charge. In our model, knowledge spillovers occur via foreign direct investments by technologically-advanced firms, so the cost is the drain of monopoly profits going abroad. Moreover, the strength of spillovers can be influenced by basic-research investments. Accordingly, the governments face the trade-off of capturing rents in the foreign country and keeping profits in the country versus realizing technology spillovers from abroad but forgoing profits in the respective sectors. Neither this trade-off, nor the way two countries will
play the ensuing basic-research investment game have been addressed in the previous literature.  

3 The Model

We build on the Schumpeterian growth model with a basic research sector. Two countries, denoted by \( H \) and \( F \), decide about their investment in basic research. In each country and each period \( t \) \((t = 1, 2, ...)\) there is a continuum of identical households of measure \( L_j, j \in \{ H, F \} \) that enjoy strictly increasing utility in consumption \( u(c) \), inelastically supply one unit of labor, and receive an equal share of the profits made by the final-good firm and from intermediate goods production. Throughout the paper we use \( j, k \in \{ H, F \} \). If both are used, we always assume \( j \neq k \). For each country and each period we consider a government maximizing the well-being of its citizens by publicly providing basic research that is financed by an income tax. Accordingly, we employ a non-overlapping generations model in which each generation elects a government to provide public goods (here basic research) to maximize its welfare. We first describe the production side of the economy and derive the equilibrium for a given level of basic research for each country. Then we study the basic research game played by the countries.

3.1 Production

In this section we describe the production side of the economy for a particular country \( j \) in a typical period \( t \).

3.1.1 Final-good sector

In the final-good sector, a continuum of competitive firms produces the homogeneous consumption good \( Y \) according to

\[
Y_j = L_j^{1-\alpha} \int_0^1 [A(i)x_j(i)]^\alpha di.
\]

\(^4\)Gersbach et al. (forthcoming) study how openness affects the incentives to invest in basic research in a single-country model with a given world technology frontier. In this paper, we study how two countries strategically interact with their basic-research investments, thereby determining the technology frontier.

\(^5\)More precisely, we make the standard assumptions \( u'(c) > 0, u''(c) < 0 \).

\(^6\)This is equivalent to maximizing the consumption of the current generation.
There is a continuum of varieties \([0, 1]\), \(x_j(i)\) stands for the amount of intermediate input of variety \(i\), and \(A(i)\) is this variety’s productivity factor. The parameter \(\alpha \in (0, 1)\) determines the output elasticity of the intermediate goods. The price of the final consumption good is normalized to one. In the following, we will operate with one representative final-good firm in each country \(j\). This firm maximizes its profit, denoted by \(\pi^y_j\),

\[
\max_{\{x_j(i)\}_{i=0}^{L^d_j}} \left\{ \pi^y_j = Y_j - \int_0^1 p_j(i)x_j(i) \, di - w_j L^d_j \right\},
\]

where \(p_j(i)\) is the price of good \(i\), \(w_j\) is the wage level, and \(L^d_j\) labor demand. Maximizing \(\pi^y\) with respect to \(x_j(i)\) and taking \(p_j(i)\) as given yields the demand functions for the intermediate goods

\[
x_j(i) = \left( \frac{\alpha A(i)^\alpha}{p_j(i)} \right)^{\frac{1}{1-\alpha}} L^d_j
\]

and the inverse demand function of labor

\[
w_j = (1 - \alpha) \left( L^d_j \right)^{-\alpha} \int_0^1 [A(i)x_j(i)]^\alpha \, di.
\]

Market clearing in the labor market implies \(L^d_j = L_j\), and \(L_j\) will be used in the following.

### 3.1.2 Intermediate-goods sectors

The intermediate goods \(x(i)\) are produced via a one-to-one technology from the final good. The intermediate firms compete à la Bertrand in their intermediate sector. The productivity leader is able to establish a monopoly position, and if there is no technological leader perfect competition prevails. Accordingly, the intermediate firms are either monopolistic or fully competitive. Their prices are denoted by \(p^c(i)\) and \(p^m(i)\), respectively. A competitive intermediate firm sets prices equal to the marginal costs. As the price of the final good has been normalized to 1, we have \(p^c(i) = 1\), and profits vanish. The monopolistic intermediate producer chooses \(p^m(i) = \frac{1}{\alpha}\). For the monopolist this leads to profits of

\[
\pi^m_j(i) = n L_j A(i)^{\frac{\alpha}{1-\alpha}},
\]

where \(n = \frac{1-\alpha}{\alpha} \alpha^{1-\alpha}\).
3.2 Technological state, innovation, and foreign entry

We assume that the world technological frontier is determined by two industrial countries, e.g. the U.S. and Europe/Japan. The productivity levels of a variety $i$ produced in the countries $H$ and $F$ in period $t$ are denoted by $A^H_t(i)$ and $A^F_t(i)$, respectively. At the end of period $t-1$, a sector $i$ in country $H$ has achieved the technological level $A^H_{t-1}(i)$. For each type of intermediate, an innovation may take place at the beginning of each period. For all varieties $i$, the innovation probability is denoted by $\rho_{jt}$. If an innovation takes place in sector $i$ in period $t$, productivity increases according to

$$A_t(i) = \gamma A_{t-1}(i)$$

with $\gamma > 1$. The innovation probability can be influenced by basic-research investments by the government. With respect to basic research, we take a lab-equipment approach and assume diminishing returns on basic-research investments. In particular, an innovation probability of $\rho_{jt}$ requires investments in period $t$ of

$$R_{jt}(\rho_{jt}) = \rho_{jt}^2 \frac{L_j \bar{A}_{t-1}}{2\theta_j}$$

where $\theta_j$ is a parameter that captures the efficiency of basic-research investments in country $j$. In our standard set-up, we interpret $\theta_j$ as the average (per capita) level of human capital in country $j$. This specification implies that the costs of basic research decline with a country’s per capita level of human capital. In Appendix B.3, we discuss the case where the costs of basic research decline with the absolute level of human capital. $\bar{A}_{t-1} = \frac{1}{t-1} \int_0^{t-1} A_{t-1}(i) di$ is an index of the average technological level in the world. With respect to the cost function of research, two issues are worth noting. First, by multiplying by the size of the country’s population we avoid a strong scale effect in the countries’ growth rates. Second, the higher the knowledge stock is the more difficult it becomes to innovate because costs of basic research increase with the average technology level $\bar{A}_{t-1}$. In this sense, the model features negative intertemporal externalities of knowledge production.

We assume for simplicity that a new innovation obtains a patent that expires after one period.\footnote{As a period it is plausible to think of roughly 20 calendar years representing the period of one generation.} Further, we assume that foreign intermediate firms enter a domestic market if they have higher productivity than domestic producers.\footnote{We exclude foreign firms contesting domestic markets if they have the same level of productivity. This can be justified by small entry costs preventing the foreign firm from entering the market.} It immediately
follows that \( A^H_t(i) = A^F_t(i) \forall i, \forall t \). Consequently, in each period there are four possible constellations for the market structure in the market for variety \( i \):

- (I): domestic monopoly in \( H \), domestic monopoly in \( F \)
- (II): domestic monopoly in \( H \), foreign monopoly in \( F \)
- (III): foreign monopoly in \( H \), domestic monopoly in \( F \)
- (IV): perfect competition in \( H \), perfect competition in \( F \)

where \textit{domestic monopoly} means that an innovator in country \( j \) possesses a patent on the highest-quality intermediate good \( i \) in country \( j \), whereas a \textit{foreign monopoly} would exist if the patent were held by an innovator headquartered in country \( k \). As patents expire after one period, a sector is characterized by perfect competition in period \( t \) when neither in country \( j \) nor in country \( k \) an innovation in this sector occurred in this period.

### 3.3 The households’ and the government’s problem

In our basic model, we intentionally keep the households’ problem extremely simple. In fact, each household is assumed to offer one unit of labor inelastically to the labor market. They receive income from working and profits as owners of firms in intermediate sectors and from final-good producers headquartered in their country.

This allows us to move immediately to the government’s problem. To establish the latter, we next derive total consumption in a country \( j \) in a period \( t \).

We start by reconsidering the expected final-good production, which writes

\[
Y_{jt} = L^1_{j-\alpha} \left[ \int_0^1 [1 - (1 - \rho_{jt})(1 - \rho_{kt})][A_{t-1}(i)]^\alpha \gamma^\alpha (x^m_j(i))^\alpha di \right. \\
+ \left. \int_0^1 (1 - \rho_{kt})(1 - \rho_{jt})[A_{t-1}(i)]^\alpha (x^c_j(i))^\alpha di \right].
\]

The first integral represents the part of final-good production resulting from the sectors where an innovation has taken place. In these sectors either a foreign innovator has entered the intermediate-good market or a domestic innovator has offered a technologically advanced product. The probability of an innovation in sector \( i \) in period \( t \) is \([1 - (1 - \rho_{jt})(1 - \rho_{kt})]\). With complementary probability \((1 - \rho_{jt})(1 - \rho_{kt})\) no innovator is successful in sector \( i \), and the technological level remains. As discussed earlier,
in sectors where no innovation occurs there is no patent protection and hence perfect competition prevails. The part of final output attributed to these sectors is reflected by the second integral. Since the innovation probabilities are not sector-specific, inserting (1) and making some minor mathematical manipulations yields

\[ Y_{jt} = L_j \left[ (1 - q_t) \gamma^{1/\alpha} \alpha^{1/\alpha} \int_0^1 [A_{t-1}(i)]^{1/\alpha} di + q_t \alpha^{1/\alpha} \int_0^1 [A_{t-1}(i)]^{1/\alpha} di \right], \]

where we use the abbreviation \( q_t \equiv (1 - \rho_{jt})(1 - \rho_{kt}) \). Using the index of the average technological level in the world \( \bar{A}_{t-1} = \int_0^1 [A_{t-1}(i)]^{1/\alpha} di \), we obtain

\[ Y_{jt} = L_j \bar{A}_{t-1}^{1/\alpha} \left[ q_t + (1 - q_t) \gamma^{1/\alpha} \alpha^{1/\alpha} \right], \]

Increasing the number of innovations in the aggregate means reducing \( q_t \). Consequently, additional innovations have a positive effect on output if and only if \( \alpha^{1/\alpha} \gamma^{1/\alpha} > 1 \), which is equivalent to \( \gamma > 1/\alpha \). This illustrates the trade-off associated with innovations concerning final-good production. On the one hand, higher quality of an intermediate good involves higher productivity in final-good production reflected by \( \gamma \).

On the other, it induces monopoly distortions in the intermediate-good market that lead to a mark-up on the price of intermediates of \( 1/\alpha \) and consequently have a negative effect on final output. If \( \gamma > 1/\alpha \), the effect of higher productivity dominates, and innovations in period \( t \) have a positive effect on final output in \( t \). However, if \( \gamma < 1/\alpha \), output in \( t \) declines as a consequence of an innovation because the monopoly distortions dominate. In the following, we assume that \( \gamma > 1/\alpha \), i.e. that innovations in \( t \) positively affect output in the same period.

Now we turn to the expected costs of producing the intermediates used in final-good production. Making use of (1), we denote the aggregate production costs of intermediate goods by \( X_{jt} \) and obtain

\[ X_{jt} = \int_0^1 (1 - q_t) x_{jt}^m(i) di + \int_0^1 q_t x_{jt}^c(i) di \]

\[ = \int_0^1 (1 - q_t) \gamma^{1/\alpha} \alpha^{1/\alpha} L_j [A_{t-1}(i)]^{1/\alpha} di + \int_0^1 q_t \alpha^{1/\alpha} L_j [A_{t-1}(i)]^{1/\alpha} di \]

\[ = L_j \bar{A}_{t-1}^{1/\alpha} \left[ q_t + (1 - q_t) \gamma^{1/\alpha} \alpha^{1/\alpha} \right], \]

where \( x_{jt}^m(i) = L_j (\alpha^2 \gamma^\alpha A_{t-1}^\alpha)^{1/\alpha} \) and \( x_{jt}^c(i) = L_j (\alpha A_{t-1}^\alpha)^{1/\alpha} \). In the last line we encounter a tradeoff associated with innovations concerning the number of intermediates.
This tradeoff is similar to the one identified with respect to final output. On the one hand, higher quality intermediate attracts higher demand (reflected by $\gamma \frac{1}{1-\alpha}$). On the other, it is protected by a patent, so supply decreases relative to the competitive situation (represented by $\alpha \frac{1}{1-\alpha}$). Consequently, if and only if $\gamma \frac{1}{1-\alpha} > \alpha \frac{1}{1-\alpha}$ the demand effect is dominant, and the amount of an innovative intermediate used in final-good production increases.

Total expected profits accruing in the intermediate sectors in country $j$ read

$$\pi_{jt} = \int_0^1 (1 - q_t) n L_j \left[ A_{t-1}(i) \right] \frac{1}{1-\alpha} \gamma \frac{1}{1-\alpha} di = (1 - q_t) n L_j A_{t-1} \gamma \frac{1}{1-\alpha}$$.

The profits in an innovative sector are $\bar{\pi}_{jt} = n L_j A_{t-1} \gamma \frac{1}{1-\alpha}$. Note that up to this point we have not said anything about the distribution of the profits to domestic or foreign innovators. This however will play a key role for the total level of consumption in a country. Given $\rho_{kt}$, expected aggregate consumption in country $j$ amounts in period $t$ to

$$C_{jt} = Y_{jt} - X_{jt} - \rho_{kt}(1 - \rho_{jt})\bar{\pi}_{jt} + \rho_{jt}(1 - \rho_{kt})\bar{\pi}_{kt} - R_{jt}.$$

The first two terms reflect net output of the final good. The third term captures the profits that innovators of country $k$ earn in country $j$, while the fourth term represents the profits innovators of country $j$ earn in country $k$. Finally, the government has to finance basic research. We assume that basic research is financed by an income tax. For simplicity, we have not explicitly written the tax into the formula for total consumption. Using the expressions above, expected aggregate consumption in country $j$ in period $t$ can be written as

$$C_{jt} = L_j A_{t-1} \left[ y_n(q_t) - \gamma \frac{1}{1-\alpha} n \left( \rho_{kt}(1 - \rho_{jt}) - \rho_{jt}(1 - \rho_{kt}) \frac{L_k}{L_j} \right) - \frac{\rho_{jt}^2}{2\theta_j} \right],$$

where $y_n(q_t) \equiv \alpha \frac{1}{1-\alpha} y(q_t) - \alpha \frac{1}{1-\alpha} x(q_t)$ represents net final-good production – i.e., total production net of the costs for the intermediate products.

### 4 Decentralized Basic-Research Investment

First we consider the static game of two governments maximizing current domestic consumption by choosing the level of basic-research investments and taking the investments of the other country as given. This can be interpreted as maximizing $C_{jt}$ via
control \( \rho_{jt} \) given \( \rho_{kt} \) rather than via \( R_{jt} \). As this problem is static, we neglect time indices in this section. We obtain the first-order condition

\[
y_n' \frac{dq}{d\rho_j} + \tilde{\gamma}n(\rho_k + (1 - \rho_k)L) - \frac{\rho_j}{\theta_j} = 0, \tag{3}
\]

where \( \tilde{\gamma} = \gamma^{\frac{\alpha}{1-\alpha}} \) and \( y_n' \equiv y_n'(q) = \alpha^{\frac{\alpha}{1-\alpha}}y'(q) - \alpha^{\frac{1}{1-\alpha}}x'(q) \). Let \( y_n^p \equiv -y_n' \). Note that \( y_n^p \) and \( y_n' \) are constants that only depend on the parameters \( \alpha \) and \( \gamma \). Further, we use \( L \equiv \frac{L_k}{L_j} \) to denote relative population size. Accordingly, we can write the reaction function of country \( j \) as

\[
\rho_j^r(\rho_k) = \theta_j \left[ (1 - \rho_k)y_n^p + \tilde{\gamma}n\left((1 - \rho_k)L + \rho_k\right) \right]. \tag{4}
\]

In the reaction function, the first term in brackets reflects the effect of a marginal increase in basic research in country \( k \) on country \( j \)'s output, while the second term represents the change in expected net profit flows from technology exchange. The derivative of \( \rho_j^r(\rho_k) \) with respect to \( \rho_k \) writes as

\[
\rho_j' = \frac{\partial \rho_j^r(\rho_k)}{\partial \rho_k} = -\theta_j \left[ y_n^p + \tilde{\gamma}n(L - 1) \right]. \tag{5}
\]

An intuitive interpretation of (5) is that a marginal increase in \( j \)'s basic-research investment is less valuable in increasing total output in country \( j \), the higher basic-research investment in country \( k \) is due to increased research duplication. Further, the effect of a marginal increase in basic research on profit flows obtained from the foreign country declines with the foreign country’s basic research efforts (reflected by \( -\theta_j\tilde{\gamma}nL \)). However, the profits prevented from flowing into the foreign country increase with the foreign country’s basic-research investment (reflected by \( \theta_j\tilde{\gamma}n \)). We also observe in (5) that the level of human capital of a country \( \theta \) affects both the reaction functions’ ordinate intercepts and their slopes. The same is true of relative population size \( L \). However, the abscissa intercepts of the reaction functions are independent of \( \theta \). Before examining the slopes of the reaction functions, we state

**Lemma 1**

If \( \gamma > \frac{1}{\alpha} \), then \( y_n^p > \tilde{\gamma}n \).

---

\(^{10}\) \( x'(q) \) and \( y'(q) \) stands for \( \frac{dx(q)}{dq} \) and \( \frac{dy(q)}{dq} \) respectively. Further,

\[
y'(q) = 1 - \gamma^{\frac{\alpha}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}}, \\
x'(q) = 1 - \gamma^{\frac{\alpha}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}},
\]

and thus \( y_n' = \alpha^{\frac{1}{1-\alpha}} (1 - \gamma^{\frac{\alpha}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}}) - \alpha^{\frac{1}{1-\alpha}} (1 - \gamma^{\frac{\alpha}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}}) \).
The proof can be found in the Appendix.

The lemma implies that we can neglect cases where \( y_p < \tilde{\gamma} n \) because we have assumed that \( \gamma > \frac{1}{\alpha} \). This allows us to determine the signs of the slopes in the countries’ reaction functions:

**Proposition 1**

*Basic research expenditures in the two countries are strategic substitutes – i.e., \( \rho'_H < 0 \) and \( \rho'_F < 0 \).*

In the remainder of the paper, we restrict the parameter set of our analysis as follows:

**Assumption 1**

\[
\theta_j(\tilde{\gamma}nL + y_p^n) < 1, \text{ which is equivalent to } \rho'_j(0) < 1.
\]

Assumption 1 requires that a country chooses basic-research spending to obtain \( \rho < 1 \) (instead of the corner solution \( \rho = 1 \)) when there are no basic-research investments by the other country. In other words, Assumption 1 says that the basic-research investments of a country cannot with certainty lead to innovation in each intermediate sector, which seems very realistic.

### 4.1 Equilibrium

In the equilibrium analysis, we show that there is a unique equilibrium \( (\rho^e_H, \rho^e_F) \in (0,1)^2 \).

**Proposition 2 (existence of unique equilibrium)**

*Given Assumption 1, there exists a unique equilibrium \( (\rho^e_H, \rho^e_F) \in (0,1)^2 \) that is characterized by*

\[
\rho^e_j = \frac{\frac{1}{\varphi} (y_p^n + \tilde{\gamma}nL) + (y_p^n + \tilde{\gamma}n/L)(\tilde{\gamma}n(1 - L) - y_p^n)}{y_j \varphi - (\tilde{\gamma}n(1 - L) - y_p^n)(\tilde{\gamma}n(1 - 1/L) - y_p^n)}. \tag{6}
\]

The proof is given in the Appendix.

### 4.2 The role of human capital

In this section we examine how basic-research investments are affected by a change in a country’s research capacity \( \theta \), which we also interpret as a country’s human capital level. The following proposition gives the results:
Proposition 3 (comparative statics with respect to $\theta$)

(i) If $L = 1$, then the country with the higher research capacity will invest more in basic research – i.e., $\rho_j^e > \rho_k^e$ if and only if $\theta_j > \theta_k$.

(ii) In equilibrium, basic-research investments of country $j$ will increase with $\theta_j$ and decrease with $\theta_k$, i.e.

$$
\frac{d\rho_j^e}{d\theta_j} > 0, \quad \frac{d\rho_k^e}{d\theta_j} < 0.
$$

The proof can be found in the Appendix.

Intuitively, if the research capacity of one country, say $\theta_j$, increases, its basic-research investments will become more productive. Hence, country $j$ will increase its basic research efforts. The reaction function of country $k$ is not directly affected by a change in $\theta_j$ but indirectly via the induced change in $\rho_j$. As according to Proposition 1 basic-research investments are strategic substitutes, $k$ will decrease $\rho_k$ in response to the increase in $\rho_j$. We illustrate this result with the following example, where both countries are of equal size $L = 1$, $\alpha = 0.5$, $\gamma = 2.1$, and $\theta_k = 0.03$. In this setting, we vary $\theta_j$ from 0.005 to 0.07. The result is shown in Figure 1.

![Graph showing $\rho_j$ and $\rho_k$ depending on $\theta_j$ given $\theta_k = 0.03$.](image)

Figure 1: $\rho_j$ (blue) and $\rho_k$ (red) depending on $\theta_j$ given $\theta_k = 0.03$.

\footnote{Note that the assumption $\gamma > 1/\alpha$ defines a critical value of $\gamma$ that depends negatively on $\alpha$. We have chosen $\alpha = 0.5$ and $\gamma = 2.1$. The latter is larger by 0.1 than the critical value. For general innovations, this value of $\gamma$ may seem relatively high, but it is justified for basic research, which if successful typically implies large technological improvements. The level of human capital $\theta_k = 0.03$ has been specified to obtain realistic growth rates.}
4.3 The role of relative population size

Now we study how countries of different sizes interact with respect to basic-research investment.

**Proposition 4 (comparative statics with respect to relative population sizes)**

(i) Let $\theta_j = \theta_k$. Then the smaller country will invest more in basic research than the larger one – i.e., $\rho^e_j > \rho^e_k$ if and only if $L_k > L_j$.

(ii) $\rho^e_j$ increases and $\rho^e_k$ decreases with $L$.

The proof can be found in the Appendix.

Recall that the relative populations of the countries are defined by $L \equiv \frac{L_k}{L_j}$. Inspection of the first-order condition of the government, (3), reveals that a relatively larger foreign country $k$ will increase the ratio of profits received from abroad and the profits paid to the foreign country. By contrast, the relation of market sizes does not play a direct role for a country’s final output and research costs. Consequently, a relatively larger market abroad makes innovation and thus basic-research investment more attractive. This finding may provide an explanation for Switzerland’s high basic-research expenditures. Figure 2 uses the parameter values of the previous example, but here, instead of varying $\theta_j$, we assume that $\theta_j = \theta_k = 0.03$ and vary the relative market size $L$.

![Figure 2: $\rho_j$ (blue) and $\rho_k$ (red) depending on $L$.](image-url)
5 Dynamics

Note that the optimal decisions by the governments are independent of the level of technology $A_{t-1}(i)$. Hence, the governments will not change their basic-research investments over time. We can infer from Equation (2) that this implies that consumption grows at the rate of the average technological level. Further, the economy does not exhibit transitional dynamics. To determine the economies’ growth rate, we can write the world’s average technological level as

$$
\bar{A}_t = \int_0^1 [1-q] \tilde{\gamma} [A_{t-1}(i)]^{\alpha/\alpha} \, di + \int_0^1 q[A_{t-1}(i)]^{\alpha/\alpha} \, di
$$

Consequently we obtain

**Proposition 5**

*The growth rate of the two economies is given by*

$$
g = (\tilde{\gamma} - 1)(1-q).
$$

It will now be interesting to establish how the growth rate reacts to changes in $\theta_j$ and $L$. The expression $\tilde{\gamma} - 1$, which is positive since $\gamma > 1$, reflects the innovation steps of a successful invention and is independent of $\theta_j$ and $L$. As a consequence, we focus on the term $1 - q$, which can be rewritten as $\rho_j + \rho_k - \rho_j\rho_k$. This expression reveals nicely that the growth rate increases with the sum of basic-research investment $\rho_j + \rho_k$ but declines with the amount of research duplication $\rho_j\rho_k$. In general, we can state

**Proposition 6**

*A higher level of human capital in one country will lead to higher growth if and only if*

$$
-\frac{d\rho_j^e}{d\theta_j} / \frac{d\rho_k^e}{\theta_j} > \frac{1 - \rho_j^e}{1 - \rho_k^e}. \tag{7}
$$

*An increase in $L$ will involve higher growth if and only if*

$$
-\frac{d\rho_j^e}{dL} / \frac{d\rho_k^e}{dL} > \frac{1 - \rho_j^e}{1 - \rho_k^e}. \tag{8}
$$

A proof of Proposition 6 follows directly from taking the derivative of the growth rate $g$ with respect to $\theta_j$ and $L$ respectively. Conditions (7) and (8) simply state that the effect of a change in $\theta_j$ and $L$ on aggregate basic-research investments is larger than the effect on duplication. With respect to a change in $\theta_j$, we obtain the following corollary:
Corollary 1

Given Assumption 1, total basic research expenditures will increase if $\theta_j$ becomes larger, i.e. $-\frac{d\rho_j^e}{d\theta_j} / \frac{d\rho_k^e}{d\theta_j} > 1$.

The proof can be found in the Appendix.

The intuition is that if $\theta_j$ increases, $\rho_j$ will become larger, while $\rho_k$ decreases. The latter effect results from the fact that the countries’ basic-research investments are strategic substitutes (cf. Proposition 1). The decline in $\rho_k^e$ induced by an increase in $\theta_j$ is a second-order effect that cannot neutralize the increase in $\rho_j^e$ with respect to aggregate innovation probability.

Further, it follows directly from Proposition 6 that the total effect of an increase in $\theta_j$ on the growth rate is positive if $\rho_j > \rho_k$. Intuitively, in this case an increase in $\theta_j$ will not only increase aggregate basic-research investments but also lead to a more unequal distribution of investments across countries, thereby reducing the duplication effect (which is largest for $\rho_j = \rho_k$). As a consequence, an increase of $\theta_j$ will positively affect the growth rate. This is different if $\rho_j < \rho_k$. Then there exists a tradeoff between the effect on aggregate basic research and duplication. Though analytically not excludable, in none of our numerical simulations could we find a case where the duplication effect dominates and leads to negative effects of an increase in $\theta_j$ on growth. The left panel of Figure 3 shows the typical situation where the growth rate increases with $\theta_j$.

![Figure 3: Growth rates depending on human capital in country $j$ (left) and for different relative population sizes $L$ (right).](image)

According to our simulations, the growth rate exhibits a U-shaped form when we vary relative population size. Numerically, we also find that the aggregate basic-research expenditures are U-shaped in $L$. Intuitively, this means that the effect of a relative increase in the population size on aggregate basic-research investments tends to be larger,
the more unequal the population shares are. Additionally, with more unequal relative basic-research investments, the negative duplication effect is lower, thus reinforcing the U-shape of the growth rate in $L$. The right hand panel of Figure 3 illustrates the typical situation.

### 6 Coordinated Basic Research Investments

Now we ask which levels of basic research the countries should choose, if they coordinate to maximize current aggregate consumption $C_t = C_{jt} + C_{kt}$. Using (2), the objective can be written as

$$C_t = Y_t - X_t - R_t = A_{t-1} \left[ (L_j + L_k)y_n(q_t) - L_j \frac{\rho_{jt}^2}{2\theta_j} - L_k \frac{\rho_{kt}^2}{2\theta_k} \right],$$

where the variables without country indices denote world values, i.e., $Y_t = Y_{jt} + Y_{kt}$ etc. Equation (9) reveals that when consumption is aggregated, the profit flows between the two countries drop out, and aggregate consumption equals net production minus total basic research expenditures.

The necessary optimality conditions for coordinated basic-research investments are

$$\rho_{jt} = \theta_j \frac{L_j + L_k}{L_j} y_n^p (1 - \rho_{kt}),$$

recalling that $y_n^p = -y_n'$. Comparing condition (10) with the reaction function of the government’s problem in the non-cooperative setting (3) reveals that the cooperative solution additionally considers the basic-research investments’ effects on the other country’s output and thus attaches weight $(L_j + L_k)$ to the increase in final-good production resulting from higher quality intermediates. The denominator $L_j$ in (10) represents the fact that in absolute terms basic research is less costly in the smaller country. Further, we observe that for a given level of basic-research investment $\rho_k$, the cooperative solution will involve higher levels of basic research in country $j$ if and only if the factor $\frac{L_j + L_k}{L_j}$ is sufficiently large to compensate for the incentive to generate higher net profit flows from technology trade in the decentralized setting [reflected by the expression $\gamma_n((1 - \rho_k)L + \rho_k)$ in Equation (3)].

From the optimality conditions (10) we obtain the optimal coordinated basic-research investments as

$$\rho_{j}^{eff} = \frac{\frac{1}{y_n}(1 + L)y_n^p - (1 + L)(1 + 1/L)(y_n^p)^2}{\frac{1}{y_n^p} - (1 + L)(1 + 1/L)(y_n^p)^2}.$$
An analytical comparison of the cooperative solution with the equilibrium values of basic research in the decentralized setting given in Proposition 2 only yields interpretable conditions for the symmetric case.

**Proposition 7**

If $L_j = L_k$ and $\theta_j = \theta_k < \frac{1}{2} \left( \frac{1}{\gamma n} - \frac{1}{y_k} \right)$, then $\rho_{j}^{eff} > \rho_{j}^{c}$ and $\rho_{k}^{eff} > \rho_{k}^{c}$.

The proof follows directly from a comparison of (11) and (6). The condition with respect to the levels of human capital $\theta$ in Proposition 7 is satisfied for reasonable parameter values. For further comparisons of the cooperative solution and the market equilibrium we make use of numerical simulations. We derive our results in the standard scenario as specified previously (i.e., $\alpha = 0.5$, $\gamma = 2.1$, $\theta_k = 0.03$) and show in Appendix B that our results are very robust. We start by holding $\theta_j = \theta_k = 0.03$ fixed and examining the effect of relative population size. We observe in Figure 4 that if the population sizes are relatively equal both countries invest too little in the decentralized equilibrium relative to the coordination optimum. However, for very different population sizes, the smaller country invests too little in basic research in the decentralized equilibrium, despite having a strong incentive due to large profit flows from the large country, while the large country invests too much. We obtain a similar result (Figure 5) when considering countries with equally large population sizes but different research capacities $\theta$: under-investment by both countries if the human capital levels are not too different, and over-investment by the country with substantially smaller levels of human capital in the case of pronounced asymmetry with respect to $\theta$.

However, even though we encounter over-investment by one country when population sizes or research capacities are very asymmetric, the total level of basic research in the cooperative solution is always higher than the one realized in the market solution. As a consequence, optimal coordination of basic research always involves a higher rate of growth (see Figures 6 and 7).

Our results could be interpreted as follows: First, consider Switzerland and the European Union. They have symmetric levels of human capital but very different market sizes. As argued earlier, the incentive to gain foreign profit flows may lead to the high basic-research investments by Switzerland. The results in this section suggest that from a cooperative perspective aiming to maximize aggregate consumption in both Switzerland and the EU, the already large Swiss basic-research investments will still be too small, while those in the EU tend to be too high. A second interesting case is
Figure 4: Basic research investments in country $j$ (left) and $k$ (right) in the decentralized solution (blue) and the coordination optimum (red) for different relative population sizes $L$.

Figure 5: Basic research investments in country $j$ (left) and $k$ (right) in the decentralized solution (blue) and the coordination optimum (red) for different levels of human capital $\theta_j$, given $\theta_k = 0.03$.

a comparison between the EU and the US. Here both market sizes and human capital levels are approximately equal. Accordingly, our welfare analysis suggests that both regions invest too little in basic research in the decentralized equilibrium.

7 Intertemporally Optimal Basic Research Investments with and without Coordination

Until now we have assumed that the governments aim at maximizing the current period’s consumption but do not consider future periods. There are good arguments for this assumption. Governments are usually appointed for a restricted period of time and focus on that period in their decisions. Also, in supranational institutions such as
the European Commission, it is rare for the decision process to look any further than 20 years into the future (which is our interpretation of the model’s period length). Nevertheless, in this section we would like to think about whether our results change substantially when the decision-makers consider longer time horizons. Accordingly, we depict the intertemporal optimization problems of the governments in the decentralized setting and the one of coordinating basic-research investments to maximize aggregate consumption. In this section, we assume that households enjoy linear utility from consumption, i.e. \( u(c) = c \). Unfortunately, the optimization problems cannot be solved analytically. Using the same parameter values as before, our simulations suggest (a) that the decentralized equilibrium and the cooperative solution are unique, and (b) that the qualitative results remain unchanged. Of course, in quantitative terms, the intertemporal solutions exhibit generally higher levels of basic-research investment.
First, we examine the non-cooperative game. Here the governments choose paths for basic-research investment rather than investments in a single period only. The government’s problem is given by

$$\max_{\{\rho_j\}_{t=0}^\infty} \sum_{t=0}^\infty \beta^t C_{jt}$$

subject to the evolution of the stock of technological knowledge

$$\bar{A}_t = \bar{A}_{t-1}[\bar{\gamma} + q_t(1 - \bar{\gamma})].$$

(12)

We assume that the discount factor $\beta$ is sufficiently small for the objective to converge to a finite value. Using standard dynamic programming arguments, we obtain the first-order condition

$$\bar{A}_{t-1} L_j \left[ y_n^j \frac{d q_t}{d \rho_{jt}} + \bar{\gamma} n (\rho_{jt} + (1 - \rho_{kt}) L) \right] - \bar{A}_{t-1} L_j \rho_{jt} \frac{d \hat{c}_{jt+1}}{d q_t} \frac{d \rho_{jt}}{d \rho_{jt}} = 0,$$

where $\hat{c}_{jt+1} = C_{jt+1}/\bar{A}_t$. Note that

$$\hat{c}_{jt+1} = L_j \left[ y_n(q_{t+1}) - \gamma \frac{\alpha}{\alpha - 1} n (\rho_{kt+1} (1 - \rho_{jt+1}) - \rho_{jt+1} (1 - \rho_{kt+1}) L) - \rho_{jt+1}^2 \frac{2}{2\theta_j} \right],$$

(13)

which will be constant in the steady state. Focusing on the steady state, the first-order condition of country $j$ reads

$$L_j \left[ y_n^j (1 - \rho_j) + \bar{\gamma} n (\rho_k + (1 - \rho_k) L) \right] - L_j \frac{\rho_j}{\theta_j} + \beta \hat{c}_j (\rho_j, \rho_k) (\bar{\gamma} - 1) (1 - \rho_k) = 0,$$

(14)

where

$$\hat{c}_j(\rho_j, \rho_k) = L_j \left[ y_n^j (q) - \gamma \frac{\alpha}{\alpha - 1} n (\rho_k (1 - \rho_j) - \rho_j (1 - \rho_k) L) - \rho_j^2 \frac{2}{2\theta_j} \right].$$

(15)

As we have mentioned, it is not possible to solve this problem analytically. Instead, we first introduce the coordinated investment problem and then discuss the simulation results obtained.

Cooperative basic-research investments maximize discounted aggregate consumption of both countries:

$$\max_{\{\rho_{jt}, \rho_{kt}\}_{t=0}^\infty} \sum_{t=0}^\infty \beta^t (C_{jt} + C_{kt})$$

Conceptually, there exists a formula that allows us to solve explicitly for basic-research investments. However, the extremely long terms only allow interpretation via simulation.
subject to (2) and (12). Using standard dynamic programming methods, the first-order conditions are given by

\[
\bar{A}_{t-1}(L_j + L_k)y_n \frac{d q_t}{d \theta_j} - \bar{A}_{t-1} L_j \rho_{jt} \frac{\rho_{jt}}{\theta_j} + \beta \hat{c}_{t+1} \frac{d \bar{A}_t}{d q_t} \frac{d q_t}{d \rho_{jt}} = 0,
\]

where \( \hat{c}_{t+1} = C_{t+1}/\bar{A}_t \). Note that the two first-order conditions imply

\[
\frac{\theta_{jt} L}{\theta_{kt} L} = \frac{\rho_{jt} 1 - \rho_{jt}}{\rho_{kt} 1 - \rho_{kt}},
\]

which can be interpreted as a condition on the cost efficiency of aggregate research expenditures. Again we focus on the steady state of the economy. The necessary conditions for a maximum can then be rewritten as

\[
(L_j + L_k)y_n^\gamma (1 - \rho_k) - L_j \frac{\rho_j}{\theta_j} + \beta \hat{c}(\rho_j, \rho_k)(\bar{\gamma} - 1)(1 - \rho_k) = 0,
\]

where

\[
\hat{c}(\rho_j, \rho_k) = (L_j + L_k)y_n(q) - L_j \frac{\rho_j^2}{2\theta_j} - L_k \frac{\rho_k^2}{2\theta_k}.
\]

In the optimality conditions for the cooperative solution, we identify two parts. The first two summands in (16) reflect static optimality if we neglect the influence of a higher knowledge stock on future outcomes. The latter is represented by the last summand.

We now turn to numerical simulations using the parameter values of our standard specification. Figures 8 and 9 give the decentralized steady-state equilibrium values of \( \rho_j \) and \( \rho_k \) (blue) compared to the cooperative solution (red) depending on relative population sizes and the level of human capital in country \( j \) given that \( \theta_k = 0.03 \).

The next Figures 10 and 11 show the sum of basic research investments and the growth rates in the decentralized equilibrium (blue) and the cooperative solution (red) depending on \( L \) and \( \theta_j \).

A comparison with the results in Section 4 indicates that basic-research investments and growth rates are higher than in the setting where only current consumption is taken into account. However, qualitatively we obtain similar functional shapes and comparisons between the decentralized and the coordinated solutions. Also, our simulations strongly suggest that the above solutions are unique. Deviating reasonably from our standard parameter values does not change the qualitative results.\(^{13}\)

\(^{13}\)The program for our simulations can be provided on request.
Figure 8: Basic research investments in country $j$ (left) and $k$ (right) in the decentralized solution (blue) and the coordination optimum (red) for different relative population sizes $L$.

Figure 9: Basic research investments in country $j$ (left) and $k$ (right) in the decentralized solution (blue) and the coordination optimum (red) for different levels of human capital $\theta_j$.

Figure 10: Total basic-research investments in the decentralized solution (blue) and the coordination optimum (red) depending on relative population sizes $L$ (left) and levels of human capital $\theta_j$ (right).
Figure 11: Growth rates in the decentralized solution (blue) and the coordination optimum (red) depending on relative population sizes $L$ (left) and levels of human capital $\theta_j$ (right).

8 Conclusions

We have developed a two-country framework to study basic-research investment in open economies. Each country faces the following trade-off: Via basic-research investments, domestic output increases and, additionally, profits can be generated by domestic firms in the foreign country. On the other hand, basic research is costly and leading-edge technology could be imported by free-riding on the other country’s basic research efforts. We examine (a) the decentralized game and (b) the cooperative solution where countries coordinate their basic-research investments.

We find that in the decentralized game, basic-research investments are strategic substitutes. A country’s basic-research investments increase with its average level of human capital and decrease with the human capital of the foreign country. Moreover, all else being equal, a small country has higher incentives to invest in basic research than a large country, because the relation between profit inflows and profit outflows is greater. This may explain the large basic-research investments undertaken by small open countries such as Korea or Switzerland. Compared with the optimal basic-research investments when countries coordinate, there may be cases where one country will invest too much in basic research in the decentralized setting if the countries’ human capital levels or market sizes are very asymmetric. However, compared to the coordination optimum the total investments in basic research are always too low in the decentralized setting. This directly implies that the global rate of economic growth will be too low if governments pursue national basic research strategies.
The paper opens up several avenues for future research. First, it would be interesting to take into account different degrees of openness, implying that only a share of firms can enter the foreign market. Considering openness as a governmental choice, it would be interesting to know how openness and basic-research investments interact. Second, the model can be extended with respect to its micro-economic foundations. For example, explicitly modeling firms’ decisions on applied research would enrich the model to capture interactions between basic and applied research. This is particularly interesting from the perspective of a global labor market with firms and governments competing for the best applied and basic researchers. Moreover, our model could be extended to explore the firms’ location and off-shoring decisions and to examine how these decisions affect the governments’ incentives to invest in basic research.
Appendix

A Proofs

A.1 Proof of Lemma 1

\( y_p^n - \tilde{\gamma}n \) can be written as

\[
y_p^n - \tilde{\gamma}n = \alpha^{\frac{\alpha}{1-\alpha}} (\tilde{\gamma}^{\frac{1}{1-\alpha}} - 1) - \alpha^{\frac{1}{1-\alpha}} (\tilde{\gamma}^{\frac{1}{1-\alpha}} - 1) - \tilde{\gamma} \frac{1-\alpha}{\alpha} \alpha^{\frac{2}{1-\alpha}}.
\]

This expression can be transformed into

\[
y_p^n - \tilde{\gamma} n = (\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}) (\tilde{\gamma}^{\frac{1}{1-\alpha}} - 1).
\]

For \( \alpha \in (0, 1) \), we obtain \( \alpha^{\frac{\alpha}{1-\alpha}} > \alpha^{\frac{1}{1-\alpha}} \). If additionally \( \tilde{\gamma}^{\frac{1}{1-\alpha}} > 1 \), which is equivalent to \( \gamma > 1/\alpha \), it follows that \( y_p^n - \tilde{\gamma} n > 0 \).

\[\square\]

A.2 Lemmata 2 and 3 with Proofs

Now we state and prove two lemmata that are useful for the following proofs. We define the function

\[
\rho_j^j(\rho_k) = \theta_j \left[ (1 - \rho_k)y_p^n + \tilde{\gamma} n \left( (1 - \rho_k)L + \rho_k \right) \right],
\]

which we allow to assume values in \( \mathbb{R} \). Note that on the interval \([0, 1]\) the function (18) is identical to the reaction function given by (4). As there will be no confusion, we will also speak of function (18) as country \( j \)'s reaction function. We denote the inverse function of \( \rho_j^j(\rho_k) \) as \( \rho_k^j(\rho_j) \). Hence, \( \rho_k^j(0) \) gives the value of \( \rho_k \) such that country \( j \)'s preferred basic-research investment is 0, i.e. that \( \rho_k^j(\rho_k) = 0 \).

Lemma 2

If \( y_p^n \geq \tilde{\gamma} n \), then \( \rho_k^j(0) > 1 \).

Proof. To verify the lemma we can write

\[
\rho_k^j(0) = \frac{\tilde{\gamma} n L + y_p^n}{\tilde{\gamma} n L + y_p^n - \tilde{\gamma} n}.
\]

The condition on \( y_p^n \) at the beginning of the lemma ensures that the denominator of (19) is positive. Then the claim in Lemma 2 follows immediately.

\[\square\]
The next lemma examines the slopes of the countries’ reaction functions. For this purpose, we use
\[
\rho^k_j(\rho_j) = \theta_k \left[ (1 - \rho_j) y_n^p + \hat{\gamma} n ((1 - \rho_j)(1/L) + \rho_j) \right],
\]
representing the reaction function of country \( k \), which is symmetric to \( \rho^j_j(\rho_k) \) as defined in (18). We refer to the inverse of \( \rho^k_k(\rho_j) \) by \( \rho^j_j(\rho_k) \).

**Lemma 3**

If \( y_n^p \geq \hat{\gamma} n \), a unique interior equilibrium \((\rho^e_j, \rho^e_k) \in (0, 1)^2 \) implies
\[
\left. \frac{d\rho^j_j(\rho_k)}{d\rho_k} \right|_{\rho_k} > \left. \frac{d\rho^k_k(\rho_j)}{d\rho_j} \right|_{\rho_j}.
\]

**Proof.** As the reaction functions are linear, it is sufficient to show that \( \rho^j_j(0) < \rho^k_k(0) \). This follows directly from Lemma 2 and Assumption 1. \( \square \)

### A.3 Proof of Proposition 2

According to Proposition 1, \( \rho'_H < 0 \) and \( \rho'_F < 0 \). It follows directly from Lemma 2 and Assumption 1 that the reaction functions intersect in \((0, 1) \times (0, 1)\). Note that Assumption 1 also prevents the two reaction functions from coinciding, which would involve multiple equilibria.

The particular equilibrium values \((\rho^e_j, \rho^e_k)\) follow directly from calculation of the intersection of the reaction functions. \( \square \)

### A.4 Proof of Proposition 3

Let us first consider (ii). According to (4), \( \frac{\partial \rho^j_j(\rho_k)}{\partial \theta_j} > 0 \forall \rho_k \in (0, 1) \). The inverse of the reaction function of \( k \), \( \rho^k_k(\rho_k) \), remains unchanged. As according to Proposition 1 we have \( \rho'_j < 0 \) and \( \rho'_k < 0 \), it follows from Lemma 3 that the new intersection of \( \rho^j_j(\rho_k) \) and \( \rho^k_k(\rho_k) \) involves a higher level \( \rho^e_j \) and a lower value \( \rho^e_k \).

Now consider item (i). Given the symmetry of the two countries’ reaction functions, we see that if \( \theta_j = \theta_k \) and \( L = 1 \), then \( \rho^e_j = \rho^e_k \). The claim in (i) can be verified by using (ii) and the following line of reasoning: The associated investment in basic research at any pair \((\hat{\theta}_j, \hat{\theta}_k)\) can be decomposed in two steps. First, we can start from the basic research levels associated with \((\theta_j = \hat{\theta}_k, \hat{\theta}_k)\), which implies symmetric basic-research
investments. Second, we can adjust basic research levels by increasing or decreasing $\theta_j$ to $\hat{\theta}_j$.  

A.5 Proof of Proposition 4

We start with (ii). As $\rho'_j < 0$ and $\rho'_k < 0$, an increase in the relative population size of country $k$ affects the reaction functions in the following way: $\frac{\partial \rho'_j(\rho_k)}{\partial L} > 0 \ \forall \rho_k \in (0,1)$ and $\frac{\partial \rho'_k(\rho_j)}{\partial L} < 0 \ \forall \rho_j \in (0,1)$. That is, after the increase in $L$, the reaction function of $j$ lies above the reaction function before the increase, and the new reaction function of $k$ lies below the old one. Since $\rho'_j < 0$ and $\rho'_k < 0$, this immediately implies that the new intersection of the reaction functions involves $\frac{d \rho'_j}{d L} > 0$ and $\frac{d \rho'_k}{d L} < 0$.

(i) Since $\rho_j = \rho_k$ if $L = 1$ and $\theta_j = \theta_k$, it follows directly from item (ii) of this proposition that $\rho_j > \rho_k$ if $L > 1$.  

A.6 Proof of Corollary 1

Consider an increase in $\theta_j$. According to (4), this involves $\frac{\partial \rho'_j(\rho_k)}{\partial \theta_j} > 0 \ \forall \rho_k \in (0,1)$, while the reaction function of $k$, $\rho'_k(\rho_k)$, remains unchanged. As a consequence, the new equilibrium will still be on $\rho'_j(\rho_k)$ and would involve no change in the sum of basic-research investments if $\frac{\partial \rho'_j(\rho_k)}{\partial \rho_k} = -1$, while $\frac{\partial \rho'_k(\rho_k)}{\partial \rho_k} < (>) - 1$ would imply higher (lower) total basic-research investment. Due to Assumption 1 and Lemma 2, we have $\frac{\partial \rho'_k(\rho_k)}{\partial \rho_k} < -1$.

B Robustness

B.1 Larger range of parameter values

In Section 6 we derived our results on the relation between the decentralized equilibrium and the planner solution by varying either relative population size while keeping both countries’ human capital levels fixed at $\theta_j = \theta_k = 0.03$ or by varying country $j$’s level of human capital while assuming symmetric population sizes. The simulation results depicted in Figures 12 through 15 show that the results derived in Section 6 possess broad validity. Moreover, the same qualitative results are obtained for a wide range of other parameter values for $\alpha$, $\gamma$, and $\theta_k$.  

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In all the figures we have included a red plane indicating the value zero in the dependent variable for ease of comparison. The first two figures (Figures 12 and 13) show the difference between basic-research investments in the cooperative optimum and in the decentralized equilibrium on the vertical axis. These simulations verify that if countries are similar with respect to population sizes and human capital, they both invest too little in basic research in the decentralized equilibrium, relative to the coordination optimum, i.e. the difference is positive. The areas where the red plane is above the blue graph indicate the parameter values of $L$ and $\theta_j$, where over-investment in basic research occurs in the decentralized equilibrium. As argued in Section 6, this is the case if the countries are sufficiently asymmetric with respect to population size and human capital.

![Figure 12: Difference between $\rho_j^{ef}$ and $\rho_j^e$ depending on $L$ and $\theta_j$, given $\theta_k = 0.03$.](image)

Figures 14 and 15 show the difference in total basic-research investment and in the growth rates between the cooperative solution and the decentralized equilibrium. The two graphs indicate that, in accordance with the results in Section 6, for all parameter values total basic-research investment is higher in the cooperative solution than in the decentralized equilibrium, which leads to correspondingly higher rates of growth.

### B.2 Coordination objective

As already discussed, the coordination of basic-research investments aiming to maximize the sum of aggregate consumption in both countries involves higher levels of basic
Figure 13: Difference between $\rho_k^{eff}$ and $\rho_k^e$ depending on $L$ and $\theta_j$ given $\theta_k = 0.03$.

Figure 14: Difference between aggregate basic research expenditure in the decentralized equilibrium and cooperative solution depending on $L$ and $\theta_j$.

research in the smaller country because in absolute terms the costs of basic research are lower there. This incentive will not be present if coordination is concerned with maximizing net per capita consumption:

$$\bar{c}_t = \bar{A}_{t-1} \left( 2y_t(q_t) - \frac{\rho_j^2}{2\theta_j} - \frac{\rho_k^2}{2\theta_k} \right).$$ (21)

The necessary conditions for an optimum are

$$\rho_j = 2\theta_j y_t^p(1 - \rho_k).$$ (22)
Figure 15: Difference between growth rates in the decentralized equilibrium and cooperative solution depending on $L$ and $\theta_j$.

Note that there is no weighting factor reflecting relative population sizes. Instead, the contribution of basic research to each country’s per capita output levels obtains the same weight in the cooperative solution. When comparing (22) with the optimality conditions of the cooperative solution in Section 6, we observe that they coincide for equal population sizes. The following figures illustrate how the decentralized equilibrium differs from the coordination optimum maximizing net per capita consumption for different relative population sizes in the standard scenario.

Figure 16: Basic research investments in country $j$ (left) and $k$ (right) in the decentralized solution (blue) and the coordination optimum (red) for different relative population sizes $L$.

Figure 16 indicates that when cooperation maximizes net per capita consumption, the
decentralized equilibrium implies too little basic-research investment in large countries and too much in small ones. The contrary was the case in the coordination optimum maximizing aggregate consumption. The conclusions that both objectives share is that there is under-investment in basic research if the countries are symmetric with respect to population size and human capital levels. However, for very asymmetric population sizes, the decentralized equilibrium involves total basic-research investments and growth rates that are too high relative to the net per-capita consumption coordination optimum.

### B.3 Different research costs

Let us now assume that the costs of basic research decline with absolute levels of human capital rather than with per capita human capital levels. Then the research costs can be written as \( R_j = \rho_j^2/(2\theta_j) \). Consequently, the governments’ objectives in the non-cooperative setting can be written as

\[
C_{jt} = L_j \bar{A}_{t-1} \left[ y_n(q_t) - \gamma^\alpha n(\rho_{kt}(1 - \rho_{jt}) - \rho_{jt}(1 - \rho_{kt})L) \right] - \frac{\rho_{jt}^2}{2\theta_j}.
\]

The reaction functions are

\[
\rho_j^r(\rho_k) = \theta_j L_j \left[ (1 - \rho_k)y_n^p + \tilde{\gamma}n((1 - \rho_k)L + \rho_k) \right].
\]

We observe that the only difference with respect to the reaction functions in Section 4 is that the optimal level of a country’s basic-research investments increases with its population size. Now there are two conflicting motives for basic-research investment.
with respect to relative population size (and given total population). On the one hand, a relatively larger foreign market increases incentives for basic research due to higher net profit flows. On the other hand, a smaller home market reduces the incentives for basic research.

Coordinating basic research to maximize current aggregate consumption \( \hat{C}_j + \hat{C}_k \) yields the following necessary conditions for an optimum:

\[
\rho_j = \theta_j(L_j + L_k)g_n^p(1 - \rho_k).
\]

Hence, the optimal levels of the coordinated basic-research investments do not depend on relative population sizes but only on the two countries’ total population. The necessary conditions are (up to a constant) equivalent to those derived for the cooperative optimum in the previous Section B.2.

The following figures illustrate the basic-research investments in the decentralized equilibrium (blue) and the cooperative solution (red) with respect to different relative population sizes \( L \) using our standard parameter values. In the simulations we assumed \( L_j + L_k = 1 \).

![Figure 18](image_url)

Figure 18: Basic research investments in country \( j \) (left) and \( k \) (right) in the decentralized solution (blue) and the coordination optimum (red) for different relative population sizes \( L \).
Figure 19: Total basic-research investments (left) and growth rates (right) in the decentralized solution (blue) and the coordination optimum (red) for different relative population sizes $L$.

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