Vertical fiscal externalities and the environment

C. Böhringer, N. Rivers, H. Yonezawa

Working Paper 16/234
March 2016

Economics Working Paper Series
Vertical fiscal externalities and the environment∗

Christoph Böhringer† Nicholas Rivers‡
Hidemichi Yonezawa§

December 2015

∗All authors acknowledge support from Environment Canada and Carbon Management Canada. Böhringer is grateful for support from Stiftung Mercator (ZentraClim). Rivers acknowledges support from the Social Science and Humanities Research Council Canada Research Chairs program. The ideas expressed here are those of the authors who remain solely responsible for errors and omissions.
†Department of Economics, University of Oldenburg, boehringer@uni-oldenburg.de
‡Graduate School of Public and International Affairs and Institute of the Environment, University of Ottawa, nrivers@uottawa.ca
§Department of Management, Technology and Economics, ETH Zürich, hyonezaw@ethz.ch
Abstract

We show that imposition of a state-level environmental tax in a federation crowds out pre-existing federal taxes. We explain how this vertical fiscal externality can lead unilateral state-level environmental policy to generate a welfare gain in the implementing state, at the expense of other states, even absent any environmental benefits. Using a computable general equilibrium model of the Canadian federation, we show that vertical fiscal externalities can be the major determinant of the welfare change following environmental policy implementation by a state government. Our numerical simulations indicate that - as a consequence of vertical fiscal externalities - state governments can reduce greenhouse gas emissions by over 20 percent without any net cost to themselves.

Keywords: fiscal externality, climate policy, federalism, computable general equilibrium.
1 Introduction

This paper considers the interaction between state and federal taxation in the context of environmental regulation. We show that a state can levy taxes on its carbon emissions and thereby generate a net economic welfare gain for the state (at the expense of the rest of the country), even absent any environmental benefits. The mechanism behind this result is that the state-level emission tax reduces the federal tax base and thus reduces the payment of federal taxes by this state; to make up the revenue, the federal tax rate must increase thereby shifting the net burden to the other states in the federation.

In the literature on fiscal federalism this mechanism is referred to as a vertical fiscal externality. While the theory on vertical fiscal externality is well established (Dahlby and Wilson, 2003), to date, analysis of vertical fiscal externalities in an environmental context is missing. We apply the theory of vertical fiscal externality to the implementation of sub-national carbon taxes in Canada. Building on a multi-province computable general equilibrium (CGE) model of the Canadian federation, we show that the vertical fiscal externality is the dominant driver of the welfare change associated with the introduction of carbon taxes by a single state: A state can fully pass on the economic cost of carbon emissions abatement to other states in the Canadian federation. Our analysis provides important insights into how vertical fiscal externalities affect the economic impacts of environmental regulation for federations such as Canada or the US where a significant proportion of environmental policy-setting occurs at the state level.

The remainder of this paper is organized as follows. In section 2, we locate our paper’s contribution into the broader literature on vertical fiscal externalities. In section 3, we develop a simple partial equilibrium model to convey the reasoning behind the results that we produce with the numerical CGE simulations for the Canadian federation. In section 4, we describe the structure and parameterization of the CGE model underlying our numerical simulations. In section 5, we explain how we decompose the general equilibrium results to gain deeper insights into the relative importance of vertical fiscal externalities. In section 6, we discuss model results. In section 7, we conclude.
2 Literature review

Classic models of fiscal federalism offer guidance for dividing government’s responsibilities between federal and state levels. The federal government is generally considered best-suited for providing pure public goods that cross state boundaries, which is the case for climate change mitigation and other transboundary environmental problems (Oates, 1999, 2001). National implementation helps to avoid a potential ‘race to the bottom’ that could occur with state implementation, since each state faces an incentive to weaken environmental policies to attract mobile factors of production from other states.

In practice, however, some sub-national governments have been active in implementing climate change policies, especially during the last decade (Rabe, 2008; Lutsey and Sperling, 2008; Williams, 2011). State implementation of climate policies raises the possibility of vertical fiscal externalities.\footnote{We interchangeably use the terms state, province, and region to refer to a sub-national government.} While vertical fiscal externalities in a non-environmental context have received significant attention—among others from Keen and Kotsogiannis (2002), Brüllhart and Jametti (2006), Dahlby and Wilson (2003), Esteller-Moré and Solé-Ollé (2001), and Devereux et al. (2007), an assessment of the importance of vertical fiscal externalities in the context of environmental regulation is missing.

Vertical fiscal externalities arise due to the shared tax bases of state and federal governments, where a new tax by a state government has implications for revenue raised by the federal government. There are two basic conditions for the creation of vertical fiscal externalities. First, there needs to be joint occupation of tax bases by the federal and state governments. As noted in Keen (1998), a vertical fiscal externality does not require formal concurrency (i.e., federal and state governments occupying the same statutory tax base) since even when the statutory tax bases are different, the economic incidence of federal and state taxes can overlap. Second, for a fiscal externality to arise, the federal government cannot respond to a new state-level tax by changing revenue or expenditure decisions in a way that discriminates against that state. Considerations of fairness and political economy generally induce
federal governments to impose similar tax rates throughout states in a federation and to divorce expenditure decisions from sources of revenue, such that this condition typically holds in policy practice.

As for other taxes and policies, vertical fiscal externalities can have important implications for environmental policy, and these - to our best knowledge - have not been explored in the literature. In this paper, we use a computable general equilibrium model to assess the importance of vertical fiscal interactions in a climate policy setting where we focus on gross welfare effects - not accounting for potential environmental benefits of emission reduction. Focusing on the environment is useful, since the motivation for environmental taxes is typically to reduce externalities, while other taxes focus on raising revenue. Thus, the policy context is qualitatively different in our setting compared to prior work on fiscal federalism. In addition to our expanding the literature on vertical fiscal externalities to address the environment, the other novel aspect of our paper is a quantitative assessment of the importance of vertical fiscal externalities. Most other papers in the literature focus on theoretical (qualitative) results or on identifying response to a tax change by another level of government, and do not quantify welfare implications of fiscal externalities. We believe that these two contributions are especially valuable given the importance of climate change as a policy issue and the recent trend towards decentralization of climate policies.

Aside from the literature on vertical fiscal externalities, our paper is related to a number of other strands of economic research. First, there is the literature on environmental federalism, summarized by Oates (2001). Most of this literature examines interjurisdictional competition for mobile factors, sometimes referred to as the ‘race to the bottom.’ Recent papers examining interjurisdictional competition and environmental regulation in federations include Kunce and Shogren (2005), Konisky (2007), and Levinson (2003). Williams (2011) compares incentive-based to command-and-control regulations in a federation, and finds that under incentive-based regulations, states are able to offload some cost by increasing regulatory stringency. Second, there is the literature on interactions between environmental policies set by multiple levels of government. For example, Böhringer and Rosendahl (2010) examine the interaction between the EU-wide emission trading system and
Member State support schemes for renewable electricity production; in a similar vein, Roth (2012) investigates interactions between federal and state-level transport regulations. Third, our paper is closely related to the literature on environmental policy design in a second-best setting (for a review see Goulder et al. (1999)).

3 Partial equilibrium model

We present a theoretical partial equilibrium model to provide guidance to the numerical findings that we produce with our more complex computable general equilibrium model.

Assume that there are $R$ identical states (or regions) in the federation, indexed by $r = 1 \ldots R$. Consider the market for a good in state $r$, which for simplicity is characterized by linear demand and supply functions. Inverse demand and supply functions are given respectively by $p_d(q) = \varepsilon_d q + q_A (\varepsilon_s - \varepsilon_d)$ and $p_s = \varepsilon_s q$, where the equilibrium in the absence of taxes is achieved at the point $A(q_A, p_A)$, as shown in Figure 1, and where $\varepsilon_s > 0$ and $\varepsilon_d < 0$ denote the slopes of the supply and demand functions.\(^3\) State index $r$ is omitted to reduce notational burden. A pre-existing federal ad valorem tax $t_f$ causes a wedge between the producer and consumer prices, as illustrated in Figure 1.\(^4\)

The federal tax is associated with two sources of welfare loss in each state. First, the deadweight loss associated with the federal tax in state $r$ is given by the area of the triangle $(ABC)$ in Figure 1:

$$DWL_r(t_f) = \frac{(\varepsilon_s - \varepsilon_d) (p_A t_f)^2}{2 ((1 + t_f) \varepsilon_s - \varepsilon_d)^2}.$$

Second, the federal tax generates revenue for the federal government, which is a

\(^3\)Without loss of generality we can assume that the supply function has no intercept which simplifies our algebra.

\(^4\)We model the federal tax as ad valorem to reflect the fact that the majority of federal taxes in most developed countries (sales, value added, and income taxes) are of this form. As noted in Dahlby and Wilson (2003), the type of federal tax matters in determining the extent of fiscal interactions between federal and state taxes.
transfer $T_r$ of funds out of the state $r$. The state welfare loss as a result of the federal
government tax revenue is given by the rectangular area ($CBHG$):

$$T_r(t_f) = t_f p_A q_A (\varepsilon_s - \varepsilon_d)^2 \left( \frac{(1 + t_f) \varepsilon_s - \varepsilon_d}{(1 + t_f) \varepsilon_s - \varepsilon_d} \right)^2.$$  

Total federal tax revenue is $\sum R T_r(t_f)$, and this must finance an exogenous level of federal government expenditure $\sum r \bar{G}_r$. We thus assume that federal government expenditure decisions are separable from revenue sources. This assumption is likely realistic for most federations, where federal government expenditures can have a redistributive effect across the states in the federation. In Appendix E, we provide empirical evidence to show that this is a conservative assumption in the setting that we examine in this paper. Because federal expenditures are constant, we can ignore them in the welfare calculation.

Now consider the introduction of a new environmental tax in state $r = 1$, with a value of $\tau_1$ levied as a specific (excise) tax, for example on the carbon content of the good. We represent the state-level environmental tax as an excise tax since this is the manner in which carbon taxes are typically calculated (as a tax on each unit of carbon contained in the fuel). The state-level environmental tax introduces an additional wedge between consumer and producer prices in state 1 as illustrated in Figure 1. With the new state-level environmental tax, the two sources of welfare loss in state 1 are affected. The deadweight loss associated with the two taxes in state 1, holding the federal tax rate constant at $t_f$, is given by triangle area ($ADE$) in Figure 1:

$$DWL_1(t_f, \tau_1) = \frac{(\varepsilon_s - \varepsilon_d)(p_A t_f + \tau_1)^2}{2 \left( (1 + t_f) \varepsilon_s - \varepsilon_d \right)^2}.$$  

$^{5}$We do not consider environmental damage in the welfare function. Our empirical application focuses on climate change policy, for which state welfare can reasonably be considered to be separable from state emissions.
The federal tax payment of state 1 is the rectangular area \((FDIJ)\):

\[
T_1(t_f, \tau_1) = t_f p_A q_A (\varepsilon_d - \varepsilon_s)^2 - 2p_A (\varepsilon_d - \varepsilon_s) \tau_1 + \varepsilon_s (\tau_1)^2 \frac{1}{(1 + t_f) \varepsilon_d - \varepsilon_s}.
\]

It is obvious that the deadweight loss in state 1 increases as a result of the implementation of the state environmental tax from the comparison of \(DWL_r(t_f)\) and \(DWL_1(t_f, \tau_1)\) in the equations above (see also Figure 1). Here, and through the rest of the paper, we refer to the change in welfare in the implementing state caused by the application of the state-level environmental tax holding the balance of federal government revenue and expenditures in that state fixed as the carbon policy effect.\(^6\)

Likewise, the revenue raised by the federal government in state 1 is reduced as a consequence of the implementation of the state excise tax. The fall in federal government revenue is a result of reduction in the producer price as well as the equilibrium quantity caused by the implementation of the state tax. The fall in federal government revenue in the implementing state generates a welfare gain from the perspective of the implementing state, by shifting the burden of federal tax revenue to other states. We refer to this as the vertical fiscal externality effect, and it can partially or completely offset the welfare loss from the increase in the deadweight loss. Figure 2 graphically sketches the fiscal externality effect and the carbon policy effect.

In order to finance an exogenous level of government expenditure, the federal government must increase the federal tax rate from \(t_f\) to \(\tilde{t}_f\) in response to the reduction in federal revenue from state 1. We can calculate the required increase in federal government tax necessary to ensure a constant level of federal revenue:

\[
R T_r(t_f) = (R - 1) T_r(\tilde{t}_f) + T_1(\tilde{t}_f, \tau_1)
\]

We can simplify the expression for \(\tilde{t}_f\) by assuming that the number of states, \(R\), is large. When \(R\) is large, the new environmental tax in state 1 has an incremental

---

\(^6\)In the multi-region computable general equilibrium model, this effect includes not only the deadweight loss illustrated here, but also the welfare change resulting from changes in the terms of trade for the implementing state.
effect on federal government revenue, such that $\tilde{f}_f \approx f_f$.\(^7\)

We can now decompose the change in welfare in state 1 following implementation of the environmental tax into a *carbon policy effect* and *fiscal externality effect*. The fiscal externality effect is equal to the reduction in tax revenue paid by the state to the federal government, which generates a welfare gain for state 1 determined by:

$$T_1(t_f) - T_1(t_f, \tau_1) = \frac{t_f \left(2p_A(\varepsilon_s - \varepsilon_d)\tau_1 - \varepsilon_s(\tau_1)^2\right)}{((1 + t_f)e_s - e_d)^2},$$

The carbon policy effect is equal to increase in the deadweight loss of state 1 as a result of the application of the state-level environmental tax:

$$DWL_1(t_f, \tau_1) - DWL_1(t_f) = \frac{(\varepsilon_s - \varepsilon_d)(\tau_1)^2 + 2p_A t_f(\varepsilon_s - \varepsilon_d)\tau_1}{2((1 + t_f)e_s - e_d)^2}.$$

As the state tax is increased, the reduction of the federal tax payment increases at a decreasing rate (concave function), whereas the increase in the deadweight loss increases at an increasing rate (convex function). Thus, when the state tax is large, the increase in the deadweight loss outweighs the reduction of the federal tax.

Lastly, we show that when the state tax is small, the fiscal externality effect dominates the carbon policy effect. The total welfare change is:

$$\left(T_1(t_f) - T_1(t_f, \tau_1)\right) - (DWL_1(t_f, \tau_1) - DWL_1(t_f)) = \frac{\left(-2\varepsilon_s t_f - (\varepsilon_s - \varepsilon_d)\right)(\tau_1)^2 + 2p_A t_f(\varepsilon_s - \varepsilon_d)\tau_1^2}{2((1 + t_f)e_s - e_d)^2}.$$

By taking the derivative with respect to $\tau_1$, we confirm that the total effect is positive when\(^8\)

$$0 < \frac{\tau_1}{p_A} < \frac{2t_f(\varepsilon_s - \varepsilon_d)}{2\varepsilon_s t_f + (\varepsilon_s - \varepsilon_d)}.$$

\(^7\)In the numerical model that follows, we do not impose the assumption of a large number of states in calculating $\tilde{f}_f$.

\(^8\)Since the state tax is an exercise tax, the size relative to original price matters rather than the level of the exercise tax itself.
whereas it is negative when
\[
\frac{\tau_1}{p_A} > \frac{2t_f(\varepsilon_s - \varepsilon_d)}{2\varepsilon_s t_f + (\varepsilon_s - \varepsilon_d)}.
\]

Figure 3 illustrates the relative importance of the carbon policy effect and the fiscal externality effect as a function of the environmental tax rate using a simple numerical model with the same form as presented above.\(^9\) In line with our analytical reasoning we observe a welfare gain when the magnitude of the state-level environmental tax is small, followed by a reduction in welfare when the state-level environmental tax becomes large enough. We take this insight to the general equilibrium analysis, where we ascertain the importance of the fiscal externality effect in a more complex and realistic setting based on empirical data.

4 Numerical general equilibrium model

To provide quantitative estimates of the effect of vertical fiscal externalities in an environmental context, we use a static multi-sector multi-region computable general equilibrium (CGE) model of the Canadian economy. The model is described in detail in Böhringer et al. (2015). Appendix C features a formal algebraic model summary including a graphical presentation of key functional forms.\(^{10}\)

The model captures characteristics of provincial (regional) production and consumption patterns through detailed input-output tables and links provinces via bilateral trade flows. Each Canadian province is explicitly represented as a region, except Prince Edward Island and the Territories, which are combined into one region. The representation of the rest of the world is reduced to import and export flows to Canadian provinces which are assumed to be price takers in international markets. To accommodate analysis of energy and climate policies the model incorporates rich detail in energy use and carbon emissions related to the combustion of fossil fuels.

\(^9\)As is common in partial equilibrium analysis of tax policy the welfare change for the tax-implementing state is captured by the change in producer and consumer surplus adjusted for changes in tax payments. The code for replicating Figure 3 is available from the authors upon request.

\(^{10}\)A complete set of model files is provided in the electronic annex to this article.
The model features a representative agent in each province that receives income from three primary factors: labour, capital, and fossil-fuel resources. Each of these sources of income is taxed by both federal and provincial governments. The representative agent in each region is endowed with a fixed supply of labour. In the sensitivity analysis, we explore the effect of assuming an upward-sloping labour supply function. Labour is treated as perfectly mobile between sectors within a region, but not mobile between regions. The representative agent in each region also has an endowment of capital, which it rents to production sectors. For our central case simulations, we adopt a specification where capital is sectorally mobile but regionally immobile - this allows us to focus on vertical fiscal externalities and ignore horizontal externalities. We explore alternative assumptions regarding capital mobility in the sensitivity analysis. There are three fossil resources specific to the respective sectors in each province: coal, crude oil, and natural gas.

The energy goods identified in the model include coal, gas, crude oil, refined oil products and electricity. Given our analysis of CO₂ emission reduction policies, this disaggregation is essential in order to distinguish energy goods by carbon intensity and the degree of substitutability. In addition, the model features major carbon-intensive non-energy industries which are potentially most affected by emission reduction policies.

Production of output in each sector and each region is by a perfectly competitive representative firm operating with constant returns to scale. Production follows a nested constant elasticity of substitution (CES) function, which captures trade-offs between inputs of capital, labor, energy, and intermediate inputs. For extractive sectors, production requires inputs of a fixed resource factor at the top level; the top level elasticity of substitution is calibrated in line with exogenous estimates on resource supply elasticities.

Bilateral trade between provinces as well as between each province and the rest-of-world is modeled using the Armington (1969) approach, which distinguishes domestic and foreign goods by origin. Each good in intermediate and final consumption is an Armington CES aggregate of the domestically produced good and a composite of imported varieties. The latter consists of imports from the rest-of-the-world and a CES aggregate of imports from other provinces. On the export side,
product differentiation between goods supplied to different markets (i.e., the domestic home province, other provinces, and the world market) is captured through a nested constant elasticity of transformation (CET) function.

Two levels of government are explicitly represented in the model. In each province, a provincial government raises revenue from taxes on outputs and inputs to production, sales to final consumers, as well as on labour, capital, and natural resource income. Tax rates are calibrated to match benchmark government revenue from the System of National Accounts. The difference between benchmark provincial government revenues and expenditures is the provincial deficit, which is kept constant throughout the simulations reflecting no change in net indebtedness for each province. Our simulations refer to the unilateral introduction of a carbon tax by a single provincial government. We thereby hold provincial government provision of public services fixed at the benchmark level. To balance the provincial government budget in the policy counterfactuals, we (endogenously) adjust lump sum transfers received from the representative agent within the province. By using lump sum transfers as the equal-yield instrument of the provincial government throughout the simulations, we can abstain from the more detailed analysis of efficiency implications associated with alternative revenue recycling strategies (Goulder et al., 1999).

In addition to the provincial governments, there is one federal government agent that serves all provinces. The federal government raises taxes from the same bases as the provincial governments: inputs to and outputs from production sectors, sales to final consumers, and labour, capital, and natural resource income. Federal tax rates, which are identical across provinces, are calculated in the benchmark to match System of National Accounts data. Real federal government expenditure in each province is held fixed at the benchmark level. The introduction of a carbon tax by a province will have an effect on federal government revenues, by changing the size of the federal government tax base. In order to keep the federal budget in balance, we endogenously adjust federal government tax rates.\footnote{We adjust all federal government tax rates by the same proportion to balance the federal budget.} It is the presence of the federal government with its equal-yield constraint (constant real expenditure and tax rates which are set endogenously to maintain expenditure), that provides the
For model parametrization, we follow the standard approach in computable general equilibrium modeling and calibrate each production function in the model to observed cost shares and exogenous estimates of substitution elasticities. Cost share data come from Canada’s System of National Accounts, using the 2006 year Statistics Canada (2006a,b). To reflect the fact that actual policy-proposals for greenhouse gas reduction are typically made for some future year, we forward-calibrate the model to a forecast 2020 benchmark data set. The forward-calibration procedure is described in detail in Böhringer et al. (2009), and uses Environment Canada projections of economic growth and energy demand. We draw elasticity estimates for each production sector from Dissou et al. (2012) as well as Okagawa and Ban (2008). Trade elasticities are based on Narayanan et al. (2012) and fossil fuel supply elasticities are related to (Graham et al., 1999; Krichene, 2002; Ringlund et al., 2008).

5 Welfare decomposition

The virtue of the general equilibrium approach to economic analysis is its comprehensive representation of market interactions. The economic impacts of policy interference quantified by CGE models thereby captures multiple direct and indirect economic responses that can both reinforce one another or work in opposite directions. A decomposition of the general equilibrium outcome can be useful to better understand the relative importance of partial equilibrium effects. For our assessment of the role of vertical fiscal externalities in environmental regulation, we present a decomposition of the overall change in welfare following introduction of a state-level carbon tax into two effects. More specifically, the welfare change in a single province implementing a unilateral carbon tax is composed of a:

**Carbon policy effect** The carbon policy effect is the effect of the carbon tax on the welfare of the representative agent in the implementing province, holding the federal government balance in that province fixed. This effect is identical to the deadweight loss associated with the state-level environmental tax in
the partial equilibrium model in Section 3, with two exceptions. First, in the general equilibrium model, the deadweight loss associated with state-level carbon taxes can exacerbate pre-existing distortionary taxes, and likewise the revenue raised from state-level carbon taxes can be used to reduce pre-existing distortionary taxes, both of which can affect the welfare calculation (Goulder et al., 1999). Second, in the general equilibrium model, adoption of a state-level carbon tax can affect the external prices facing the implementing province. Imposition of the carbon tax in a province influences prices in other provinces via changes in bilateral trade flows. The changes in external prices correspond to changes in the terms of trade which imply a secondary welfare gain or welfare loss.\footnote{The effect of unilateral carbon regulations on terms of trade has been the focus of Böhringer et al. (2014) and Böhringer and Rutherford (2002).}

**Fiscal externality effect** The fiscal externality effect relates to the balance of the federal government within the province. Keeping federal government expenditures in the province constant while the tax transfer from the province to the government decreases, the fiscal externality effect represents a welfare gain to the province. This effect is identical to the difference in federal government revenue before and after the state-level environmental tax in the partial equilibrium model in Section 3.

We implement our decomposition by building on the proposition by Böhringer and Rutherford (2002) that each region of a multi-region CGE model can be represented as a small open economy in order to separate the domestic policy effect under fixed terms of trade. Policy-induced changes in external prices can then be imposed parametrically on the small open economy variant of each model region. We extend this approach by accounting in addition for the fiscal externality effect arising in the state-federation setting. More specifically, analysis and decomposition of welfare effects proceeds as follows. We use the multi-region model to calculate the full general equilibrium effect of the carbon tax on welfare in the tax-implementing province. The welfare effect is measured as percent change in Hicksian equivalent variation (HEV) of income from the benchmark. The multi-region solution pro-
vides all information for welfare decomposition within the single-region variant. The single-region variant represents each province identically as in the multi-region model, with three exceptions. First, it treats external prices from other provinces as parametric, rather than endogenous. Second, it treats federal government tax rates as parametric, rather than endogenous. Third, it treats the portion of federal government revenue raised in other provinces as parametric, rather than endogenous. Using the single-region variant, we can parametrically impose the carbon tax, the vector of exogenous prices facing the province, federal government tax rates, and federal government income raised in other provinces. Values for these exogenous parameters are drawn from the solution to the multi-region variant of the model. Imposing only the carbon tax and the updated vector of external prices in the single-region model, while maintaining the federal government balance at benchmark levels, produces the carbon policy effect. Imposing only the federal government tax rate and revenue from other provinces, while maintaining the carbon tax and external prices at benchmark levels, generates the fiscal externality effect.\textsuperscript{13} A more formal description of our decomposition methodology is provided in Appendix D.

It is important to consider the separability of the two effects. The fiscal externality effect is a pure transfer to the implementing region from other regions, which occurs via the federal government’s budget balance within the region. Given the homothetic utility functions of the representative agents, the fiscal externality effect does not change relative prices. As a result, the fiscal externality effect is additively separable from the carbon policy effect.

6 Scenarios and results

6.1 Policy scenarios

We show the importance of fiscal externalities arising from unilateral implementation of environmental regulation by a single state in a federation. Specifically, we

\textsuperscript{13}If we simultaneously impose all the external shocks on the single-region variant we arrive at the identical solution for the respective province as calculated by the multi-region model.
quantify the welfare effects from unilateral carbon taxation by a single Canadian province. We produce results for each province separately in a series of simulations that successively consider each province as the implementing province. We use our decomposition method to assess the relative importance of the fiscal externality effect vis-à-vis the carbon policy effect.

Revenue from the unilateral carbon tax is collected by the provincial government in the implementing province. The provincial governments use lump sum transfers to the representative agents as the equal yield instrument while the federal government maintains a constant level of real expenditure by altering all federal tax rates by the same proportion.¹⁴

### 6.2 Results

#### 6.2.1 Effect of unilateral state carbon tax on regional welfare

In our central case simulations we estimate the costs of achieving a 10 percent reduction in carbon emissions within a single Canadian province which levies a sufficiently high carbon tax on the domestic use of all fossil fuels (both for intermediate use and final consumption). Welfare impacts are reported as the percentage change in Hicksian equivalent variation of income from the no-policy benchmark. It should be kept in mind that our welfare accounting abstracts from environmental benefits; we focus on the gross economic cost of emission abatement. One reason for abstracting from environmental benefits is the lack of robust cost estimates at the provincial level; another reason is that, given the global nature of the greenhouse gas externality, the benefits from greenhouse gas emission reduction at the state level can be perceived as negligible.

Table 1 illustrates the welfare effect of achieving a unilateral 10 percent reduction of carbon in a particular province. Each column in the table represents a separate scenario, in which the implementing province is identified by the col-

¹⁴ We obtain very similar results if we use lump sum transfers to balance the federal budget, in which transfers to each province are in proportion to benchmark income. Likewise, if lump sum transfers by provinces to representative agent are not available, we obtain identical results if we rebate provincial carbon tax revenue to households by reducing labour income taxes. We show the sensitivity of the results to alternative parameterization of labour supply later in the paper.
umn heading, and imposes a unilateral carbon tax to achieve the desired 10 percent reduction in its own emissions. The total welfare change in each province is calculated by simulating the unilateral policy in the multi-region variant of our CGE model. The cost to the implementing province is indicated by the shaded cell in the table, while the cost to other regions is indicated by non-shaded cells. For example, implementation of a climate policy that reduces carbon emissions in Ontario (Canada’s largest province) by 10 percent is estimated by the model to result in a welfare gain in Ontario of 0.033 percent of income ($219 million). However, the same policy (i.e., Ontario’s carbon tax) is estimated by the model to impose a welfare loss in Quebec of 0.044 percent of income ($165 million).

The total welfare cost associated with a 10 percent reduction in emissions is heterogeneous across provinces reflecting differences in economic structures which determines the ease of substituting away from carbon in production and consumption. However, the total welfare effect of the unilateral carbon policy is positive for all cases for the implementing province, suggesting that unilateral emission reduction in a province can be welfare improving from the perspective of the province even when abstracting from potential environmental benefits. On average, the model results show that welfare increases by about 0.05 percent in the implementing province as a result of unilateral implementation of a carbon tax that reduces carbon emissions by 10 percent.

The row labelled “All” in Table 1 shows that for Canada as a whole, the total cost of the unilateral carbon policy is negative in all cases. For example, the column for Ontario suggests that imposing a unilateral policy in Ontario which cuts emissions in that province by 10 percent results in a welfare loss for Canada as a whole of $468 million, or 0.028 percent of benchmark income. Thus the welfare gain we report in the implementing province is more than offset by welfare losses in other provinces.

Finally, the row “All*” in the table reports the country-wide welfare change associated with a national carbon tax that achieves the same reduction in emissions. However, a 5 percent reduction in emissions increases welfare in that province as well.
sions as the implied by the unilateral policy in each column scenario. As expected, a nation-wide policy, which exploits the cheapest opportunities for emission reductions across all regions, achieves emission cuts at a lower cost than the unilateral state-level policy.\textsuperscript{16} To continue the previous example, a national policy that achieves the same environmental benefit as a unilateral policy which cut emissions in Ontario by 10 percent would only cost $75 million, or 0.004 percent of Canadian benchmark income.

### 6.2.2 Decomposition of results

To assess the importance of the fiscal externality effect in environmental regulation we decompose the total welfare effect into two components - the fiscal externality effect and the carbon policy effect - using the small open economy single-region variant of our model. The fiscal externality effect results from the federal government budget constraint. When the implementing province applies a carbon tax to reduce carbon emissions, it affects production, sales, and income in the province, all of which are components of the federal tax base (the impacts spill over to other provinces as well). As the federal tax base shrinks in the implementing province due to the carbon tax, the federal government increases tax rates to make up its budget shortfall. Because the federal government applies the same taxes across all provinces, the burden of the increase in federal taxes falls to a large extent on other provinces. In contrast, federal expenditures are held fixed in real terms in each province. The combination of these two effects implies that the federal budget balance (revenues less expenditures) in the implementing province declines, while it increases in other provinces. This results in an income transfer into the implementing province via the federal government budget closure rule.

We show the results of the decomposition calculation in Figure 4. Each set of columns reports on identical scenarios as referred to in the columns of Table 1. In

\textsuperscript{16}There is one exception to this in the table. In Table 1, we show that a 10 percent reduction of emissions via a carbon tax in Alberta achieves carbon emission reduction more cost effectively than a national carbon tax that achieves the same total emission reduction. This is a second-best result. In particular, it is due to the low rate of benchmark tax on intermediate use of oil products in Alberta relative to other Canadian provinces. We confirm this result by repeating the simulation above, but in which we set all provincial intermediate oil product taxes to the same level in the benchmark.
each case, the fiscal externality effect is positive (as expected) and substantial in magnitude relative to the total effect of the policy. On average, the model suggests that fiscal externalities associated with unilateral implementation of a state-level carbon tax (which is high enough to reduce emissions by 10 percent) increases welfare in the implementing province by about 0.08 percent.

The carbon policy effect is the effect of domestic emission pricing on welfare, exclusive of the fiscal externality effect. It includes both the abatement cost in the implementing region (inclusive of any tax interaction effects), as well as the terms of trade effect. The carbon policy effect results in a welfare loss in the implementing region. On average, the model suggests that the carbon policy effect results in a welfare loss of about 0.03 percent for a province that implements a carbon tax large enough to reduce emissions by 10 percent. Importantly, the (typically negative) carbon policy effect is generally smaller than the (positive) fiscal externality effect at the 10 percent emission reduction level, suggesting a welfare gain associated with introduction of a moderate unilateral carbon policy.

Figure 5 decomposes the welfare effect at different levels of emission reduction stringency ranging from 0 percent to 30 percent. As laid out in Section 3, the carbon policy effect is convex in the stringency of the state-level environmental policy, while the fiscal externality effect is concave. Consequently, following introduction of a carbon policy, welfare in the implementing state increases for small emission reductions, and is reduced for large emission reductions. We thus see that a non-zero amount of emission abatement is optimal from the perspective of a province, even neglecting environmental benefits. For the central case parametrization of our model, the welfare gains to a single province are maximized for a reduction in emissions of around 10 percent, and a reduction in emissions of up to 20 percent come at no economic cost for the unilaterally abating province.

6.2.3 Ordering of state-level climate policy adoption

Our analysis thus far has been conducted in the context of a first mover on climate change. In particular, we conduct scenarios in which we estimate the welfare im-

---

17 The one outlier is Manitoba where the terms of trade effect is large enough to render the overall carbon policy effect positive.
impact of imposing a reduction in carbon emissions in one province when no other provinces have such policy. However, it is important to clarify that the fiscal externality effect we measure is independent of the ordering in which policy adoption takes place. Table 2 replicates the analysis shown above, with a twist. In each column, we now assume that every province other than the one indicated by the column header has already implemented a carbon policy which reduces its emissions by 10 percent, and show the welfare impact associated with the adoption of similar climate policy by the last province (indicated by the column header). Welfare impacts are nearly identical to those indicated in Table 1. This result is due to the fact that the absolute magnitude of the fiscal externality effect is small relative to the total federal tax revenue, such that changes in federal tax rates are minimal. As a result, the scope for a fiscal externality remains robust after several other provinces have enacted climate policy.

Although our paper is not about coalitions, we extend our analysis to consider the welfare effects associated with coalitions of implementing states in Table 3. Here, we simulate climate policy by a successively larger coalition of states. Our coalition is chosen somewhat arbitrarily, but roughly mimics the order of adoption of state level market-based climate policies in Canada over the last several years. In each column, we indicate the provincial governments that are part of the coalition—each seek to reduce carbon emissions by 10 percent. We show that as the coalition of implementing states becomes larger, the welfare gains for each implementing state become successively smaller. When the coalition becomes sufficiently large (in the table, this corresponds to the fourth column, when BC-AB-QC-ON are all members of the coalition), the welfare change of each coalition member becomes negative. This is because when the coalition is large enough, changes in the federal tax rate required to balance the federal budget in response to coalition carbon taxes become large. However, note that even in this scenario, non-coalition members would still experience a gain in welfare from joining the coalition (but the remaining members of the coalition would experience a welfare loss from this move).

The results in the table are based on individual achievement of the 10 percent emissions target in each province (i.e., without permit trade between provinces). Simulations with permit trade between provinces have been conducted and result in similar results, with a strong fiscal externality effect as shown here.
6.3 Sensitivity analysis

We conduct sensitivity analysis to investigate the relative importance of the vertical fiscal externality effect under alternative assumptions for trade responsiveness (Armington elasticities) and closures in the labour and capital markets. For the sake of brevity we limit exposition of the sensitivity analysis to a single province - Ontario - which cuts emissions by 10 percent unilaterally using a carbon tax and returns the carbon tax revenue in lump sum to the province’s representative consumer. The results of our sensitivity analysis are presented in Figure 6.¹⁹

The Armington elasticity determines the ease of substitution between domestically produced goods and goods of the same variety produced outside the province. Lower (higher) Armington elasticities increase (decrease) the scope for shifting cost of unilateral abatement to trading partners via policy-induced changes in the terms of trade. In the sensitivity analysis we double and halve the Armington elasticities. In the central case simulations, capital is assumed to be sectorally mobile, but immobile between regions. In the sensitivity analysis, we treat capital as mobile both between regions and between sectors. We test two alternative mobility assumptions: one in which capital is mobile between sectors and Canadian provinces, and one in which we treat capital as mobile not just between Canadian provinces, but also between Canada and the rest of world.²⁰

Introducing capital mobility allows the model to capture the potential for horizontal externalities. When a province imposes a tax on carbon, part of the incidence of the tax is borne by capital. To the extent that capital is mobile between regions, it can escape the burden of the tax. Mobility of capital out of a region can worsen labour productivity in a region, with negative welfare impacts. Inversely, the mobility of capital to other regions can improve productivity in those regions: a horizontal externality. This can generate a rationale for a government to reduce the stringency of environmental taxes.

We also investigate the effect of changing the labour market closure in the

¹⁹Simulations for other provinces yield qualitatively the same results. In particular, the fiscal externality effect is prominent relative to the carbon policy effect in all sensitivity cases in all provinces.
²⁰Essentially, this involves treating the return on capital as exogenous. To accommodate capital inflows or outflows, the balance of payments constraint is modified such that the change in balance of trade is required to equal the change in balance of foreign savings.
model. In the central case simulations, the consumer is endowed with a fixed supply of labour, all of which is used in production of goods. In the sensitivity analysis, we adopt a closure where a portion of the consumer’s labour supply is consumed directly by the consumer as leisure (which enters the consumer’s utility function). Consumption of leisure responds to the price of leisure (i.e., the wage rate). With elastic labor supply, the pre-existing tax on labour renders a new carbon tax more distorting, because of negative tax interaction effects between the carbon tax and the existing labour tax (Goulder et al., 1999; Parry, 1995).

Figure 6 shows the (decomposed) welfare results for alternative Armington elasticities as well as capital and labour market closures. Changing the Armington elasticities has only second-order effects. Introducing either capital mobility or an upward-sloping labour supply increases the carbon policy effect markedly. The economic reasoning behind this is straightforward: both of these changes increase pre-existing tax distortions, and as a result of the tax interaction effect, increase the deadweight loss associated with carbon taxation. When leisure is introduced into the model (while maintaining capital as immobile between regions), the fiscal externality effect decreases. In this setting, capital bears the greater incidence of the tax since it is immobile, leading to increases in the relative wage rate and more labor supply in the regulated region. The increase in labour supply increases federal government tax revenue from the regulated region, and reduces the magnitude of the fiscal externality. When capital mobility is introduced, the fiscal externality effect is increased. Capital mobility causes some capital to relocate from the regulated region to other provinces or to the rest of the world as a result of the carbon tax. Capital relocation further reduces the federal tax base in the regulated region, which increases the fiscal externality effect.

Overall, our qualitative conclusions are robust. In particular, the fiscal externality effect remains substantial in magnitude relative to the carbon policy effect.

21 We calibrate the elasticity of substitution between leisure and consumption to match estimates of uncompensated and compensated labour supply elasticities, using the method suggested in Ballard (2000) and estimates for labour supply elasticities provided in Cahuc and Zylberberg (2004).

22
7 Conclusion

Whenever a sub-national government implements a new environmental policy, there is scope for vertical fiscal externalities: some or all of the net burden of the environmental regulation may be shifted to other jurisdictions in the federation as a result of the federal budget constraint.

In this paper, we have first developed a theoretical partial equilibrium model to trace the central mechanisms of vertical fiscal externalities for state-level environmental taxation. We then have used a large-scale CGE model for the Canadian federation to show that vertical fiscal externalities are the dominant driver of the economic effects triggered by the introduction of carbon taxes at the provincial level: An individual province can reduce its carbon emissions by up to 20 percent without any welfare loss (absent any environmental benefits) by shifting the net burden to the other provinces in the Canadian federation.

Our findings may have important policy implications given the increasing decentralization of environmental regulation. Vertical fiscal externalities can become particularly relevant in the policy debate on combatting transboundary or global environmental externalities such as greenhouse gas emissions where the environmental benefits of unilateral provincial action are small or negligible. In this case, a state may no longer suffer from economic cost as a consequence of taking a fore-runner role in environmental regulation.

There are a number of ways to extend the current analysis. First, we could adopt a strategic perspective, where tax-setting by one government responds to tax choices by other governments. Second, we could examine the effect to which inter-governmental grants affect our conclusions regarding vertical fiscal externalities. Third, we could test the sensitivity of our results for alternative environmental policy instruments (e.g., energy or carbon efficiency standards). Fourth, we could calculate the optimal environmental tax in an federal setting considering alternative revenue recycling options. We plan to address these issues in future research.
References


Statistics Canada (2006a). Final demand by commodity, S-level aggregation, Table 381-0012.

Statistics Canada (2006b). Inputs and output by industry and commodity, S-level aggregation, Table 381-0012.
Statistics Canada (2015a). Estimates of population, by age group and sex for July 1, Canada, provinces and territories.


Figure 1: Partial equilibrium model setup. The model reflects the market for a good in a single state in a federation. In the absence of taxes, supply of the good is given by \( \varepsilon_s q \) and demand for the good is given by \( \varepsilon_d q + q_A (\varepsilon_s - \varepsilon_d) \). The state corresponds to state 1 in the text, which imposes a unilateral environmental excise tax. State 1’s environmental excise tax, \( \tau_1 \), interacts with an existing ad valorem federal tax, \( t_f \).
Figure 2: Welfare effect of unilateral implementation of state environmental tax in a partial equilibrium model. When a new state-level excise tax $\tau_1$ is imposed, it exacerbates the pre-existing distortion caused by the federal ad valorem tax $t_f$. This deadweight loss is given by the sum of the areas of the orange hatched triangles and the dotted rectangle. By reducing the federal tax base, the new state tax also reduces federal tax revenue in the state, which is given by the sum of the areas of the green checked rectangle and the dotted rectangle. As a result, the net welfare change following a tax is determined by comparing the area of the orange hatched triangles (carbon policy effect) with the green checkered rectangle (fiscal externality effect).
Figure 3: Numerical simulation of welfare impact of state environmental tax in a partial equilibrium model. Implementation of a new state-level environmental tax exacerbates pre-existing distortions, causing a loss in state welfare given by the solid line labeled the carbon policy effect. The new state-level environmental tax also shrinks the state tax base and reduces federal revenue raised in the state, which increases state welfare. This effect is called the fiscal externality effect and is given by the dotted line. For low levels of the state excise tax, the fiscal externality effect dominates the carbon policy effect, such that the net effect on state welfare - given by the dashed line - is positive. The numerical simulation based on stylized data is purely illustrative. Parameters corresponding to the description in Section 3 are: $\varepsilon_S = 0.75$, $\varepsilon_D = -0.5$ (and we assume that the intercept of the demand function occurs at $p = $10/unit). These parameters result in $p_A = $8/unit and $q_A = 6$ units.

We assume $t_f = 0.25$, such that the consumer price with the federal tax becomes $9.1/unit. Computer code for replicating the Figure is available from the authors upon request.
Figure 4: Decomposition of welfare effect from unilateral carbon policy implementation. Each set of three bars is an individual model simulation in which the corresponding province implements a unilateral carbon tax to reduce its own emissions by 10 percent. The carbon policy effect is the deadweight loss associated with the policy in the implementing region, and the fiscal externality effect corresponds to the balance of the federal government in the implementing region. The total welfare change is the sum of the carbon policy and fiscal externality effects. We use the following abbreviations for Canadian provinces/regions: AB - Alberta; BC - British Columbia; MB - Manitoba; NB - New Brunswick; NL - Newfoundland and Labrador; NS - Nova Scotia; ON - Ontario; QC - Quebec; SK - Saskatchewan; RC - Rest of Canada (Prince Edward Island and Territories).
Figure 5: Decomposition of welfare effect from unilateral carbon policy implementation at different emission reduction stringencies. Each panel is a separate simulation of a unilateral market based policy that reduces carbon emissions in the province by 0 to 30 percent. The total effect on welfare is decomposed into a fiscal externality effect and a carbon policy effect. We use the following abbreviations for Canadian provinces/regions: AB - Alberta; BC - British Columbia; MB - Manitoba; NB - New Brunswick; NL - Newfoundland and Labrador; NS - Nova Scotia; ON - Ontario; QC - Quebec; SK - Saskatchewan; RC - Rest of Canada (Prince Edward Island and Territories).
Figure 6: Sensitivity analysis of welfare effect from unilateral carbon policy implementation in Ontario to different model closures. In each case, Ontario reduces emissions unilaterally by 10 percent. Scenarios are as follows: bench - benchmark closure and parameters as described in the text; armin_double - double Armington elasticities; armin_half - half Armington elasticities; capital - capital is mobile between sectors and regions, including the rest of the world; capital_canada - capital is mobile between sectors and Canadian regions; leisure - the representative consumer demands leisure such that labour supply is endogenous; capital_leisure - capital is mobile between regions and the representative consumer demands leisure.
Table 1: Welfare in percent change in Hicksian equivalent variation of income (in million Canadian dollar inside of the parentheses). Welfare change is due to unilateral implementation of a 10 percent emission cut by the column-region as the first-mover. Welfare impacts are associated with the row-region and "All" indicates the welfare impact of Canada. "All*" indicates the Canadian welfare impact of the national carbon price achieving the same emission reduction as the corresponding unilateral emission cut.
<table>
<thead>
<tr>
<th></th>
<th>AB</th>
<th>BC</th>
<th>MB</th>
<th>NB</th>
<th>NL</th>
<th>NS</th>
<th>ON</th>
<th>QC</th>
<th>SK</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>0.068</td>
<td>-0.051</td>
<td>-0.023</td>
<td>-0.001</td>
<td>-0.003</td>
<td>-0.002</td>
<td>-0.207</td>
<td>-0.040</td>
<td>-0.017</td>
</tr>
<tr>
<td></td>
<td>(132)</td>
<td>(-101)</td>
<td>(-46)</td>
<td>(-2)</td>
<td>(-6)</td>
<td>(-4)</td>
<td>(-406)</td>
<td>(-77)</td>
<td>(-33)</td>
</tr>
<tr>
<td>BC</td>
<td>-0.035</td>
<td>0.052</td>
<td>-0.004</td>
<td>-0.002</td>
<td>-0.001</td>
<td>-0.001</td>
<td>-0.050</td>
<td>-0.022</td>
<td>-0.003</td>
</tr>
<tr>
<td></td>
<td>(-79)</td>
<td>(118)</td>
<td>(-8)</td>
<td>(-4)</td>
<td>(-3)</td>
<td>(-3)</td>
<td>(-113)</td>
<td>(-49)</td>
<td>(-8)</td>
</tr>
<tr>
<td>MB</td>
<td>-0.024</td>
<td>-0.017</td>
<td>0.146</td>
<td>-0.002</td>
<td>-0.001</td>
<td>-0.001</td>
<td>-0.025</td>
<td>-0.021</td>
<td>-0.009</td>
</tr>
<tr>
<td></td>
<td>(-14)</td>
<td>(-10)</td>
<td>(86)</td>
<td>(-1)</td>
<td>(-1)</td>
<td>(-1)</td>
<td>(-15)</td>
<td>(-13)</td>
<td>(-5)</td>
</tr>
<tr>
<td>NB</td>
<td>-0.011</td>
<td>-0.013</td>
<td>-0.002</td>
<td>-0.050</td>
<td>-0.006</td>
<td>-0.002</td>
<td>-0.027</td>
<td>-0.021</td>
<td>-0.001</td>
</tr>
<tr>
<td></td>
<td>(-4)</td>
<td>(-4)</td>
<td>(-1)</td>
<td>(16)</td>
<td>(-2)</td>
<td>(-1)</td>
<td>(-9)</td>
<td>(-7)</td>
<td>(0)</td>
</tr>
<tr>
<td>NL</td>
<td>-0.010</td>
<td>-0.020</td>
<td>-0.003</td>
<td>-0.030</td>
<td>0.047</td>
<td>-0.013</td>
<td>-0.087</td>
<td>-0.063</td>
<td>-0.002</td>
</tr>
<tr>
<td></td>
<td>(-2)</td>
<td>(-4)</td>
<td>(-1)</td>
<td>(-6)</td>
<td>(9)</td>
<td>(-2)</td>
<td>(-16)</td>
<td>(-12)</td>
<td>(0)</td>
</tr>
<tr>
<td>NS</td>
<td>-0.018</td>
<td>-0.020</td>
<td>-0.004</td>
<td>-0.018</td>
<td>-0.007</td>
<td>-0.012</td>
<td>-0.062</td>
<td>-0.029</td>
<td>-0.004</td>
</tr>
<tr>
<td></td>
<td>(-7)</td>
<td>(-8)</td>
<td>(-2)</td>
<td>(-7)</td>
<td>(3)</td>
<td>(-5)</td>
<td>(-25)</td>
<td>(-12)</td>
<td>(-2)</td>
</tr>
<tr>
<td>ON</td>
<td>-0.025</td>
<td>-0.029</td>
<td>-0.009</td>
<td>-0.005</td>
<td>-0.004</td>
<td>-0.003</td>
<td>0.025</td>
<td>-0.038</td>
<td>-0.006</td>
</tr>
<tr>
<td></td>
<td>(-167)</td>
<td>(-192)</td>
<td>(-61)</td>
<td>(-32)</td>
<td>(-24)</td>
<td>(-20)</td>
<td>(165)</td>
<td>(-252)</td>
<td>(-38)</td>
</tr>
<tr>
<td>QC</td>
<td>-0.016</td>
<td>-0.017</td>
<td>-0.007</td>
<td>-0.007</td>
<td>-0.003</td>
<td>-0.002</td>
<td>-0.045</td>
<td>0.040</td>
<td>-0.003</td>
</tr>
<tr>
<td></td>
<td>(-60)</td>
<td>(-65)</td>
<td>(-26)</td>
<td>(-28)</td>
<td>(-10)</td>
<td>(-6)</td>
<td>(-170)</td>
<td>(151)</td>
<td>(-13)</td>
</tr>
<tr>
<td>SK</td>
<td>-0.010</td>
<td>-0.009</td>
<td>-0.007</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>-0.051</td>
<td>-0.011</td>
<td>0.016</td>
</tr>
<tr>
<td></td>
<td>(-5)</td>
<td>(-5)</td>
<td>(-3)</td>
<td>(0)</td>
<td>(0)</td>
<td>(0)</td>
<td>(-27)</td>
<td>(-6)</td>
<td>(9)</td>
</tr>
<tr>
<td>RC</td>
<td>0.010</td>
<td>0.004</td>
<td>0.011</td>
<td>-0.020</td>
<td>-0.006</td>
<td>-0.010</td>
<td>0.003</td>
<td>-0.018</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(1)</td>
<td>(2)</td>
<td>(-3)</td>
<td>(-1)</td>
<td>(-1)</td>
<td>(0)</td>
<td>(-3)</td>
<td>(1)</td>
</tr>
<tr>
<td>All</td>
<td>-0.012</td>
<td>-0.016</td>
<td>-0.004</td>
<td>-0.004</td>
<td>-0.002</td>
<td>-0.003</td>
<td>-0.037</td>
<td>-0.017</td>
<td>-0.005</td>
</tr>
<tr>
<td></td>
<td>(-205)</td>
<td>(-270)</td>
<td>(-59)</td>
<td>(-67)</td>
<td>(-40)</td>
<td>(-42)</td>
<td>(-616)</td>
<td>(-279)</td>
<td>(-89)</td>
</tr>
<tr>
<td>All*</td>
<td>-0.013</td>
<td>-0.002</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>-0.007</td>
<td>-0.002</td>
<td>-0.001</td>
</tr>
<tr>
<td></td>
<td>(-214)</td>
<td>(-27)</td>
<td>(-4)</td>
<td>(-5)</td>
<td>(-3)</td>
<td>(-6)</td>
<td>(-125)</td>
<td>(-36)</td>
<td>(-15)</td>
</tr>
</tbody>
</table>

Table 2: Welfare change in percent change in Hicksian equivalent variation of income (in million Canadian dollar inside of the parentheses). Unlike Table 1, welfare change is due to unilateral implementation of a 10 percent emission cut by the column-region as the last-mover instead of the first-mover. Specifically, welfare impact is calculated when all provinces except the province in the column-region implement emission cut as well as when all provinces implement emission cut, and the difference is shown as the welfare change in this table. Welfare impacts are associated with the row-region and "All" indicates the welfare impact of Canada. "All*" indicates the Canadian welfare impact of the national carbon price achieving the same emission reduction as the corresponding unilateral emission cut.
Table 3: Welfare in percent change in Hicksian equivalent variation of income (in million Canadian dollar inside of the parentheses). Welfare change is due to the implementation of a 10 percent emission cut by the column-region(s). Welfare impacts are associated with the row-region and "All" indicates the welfare impact of Canada. "All*" indicates the Canadian welfare impact of the national carbon price achieving the same emission reduction as the corresponding emission cut.
C  Algebraic model summary

The model is formulated as a system of nonlinear inequalities. The inequalities correspond to the three classes of conditions associated with a general equilibrium: (i) exhaustion of product (zero profit) conditions for constant-returns-to-scale producers, (ii) market clearance for all goods and factors and (iii) income-expenditure balances. The first class determines activity levels, the second class determines prices and the third class determines incomes. In equilibrium, each of these variables is linked to one inequality condition: an activity level to an exhaustion of product constraint, a commodity price to a market clearance condition and an income to an income-expenditure balance. Constraints on decision variables such as prices or activity levels allow for the representation of market failures and regulation measures. These constraints go along with specific complementary variables. In the case of price constraints, a rationing variable applies as soon as the price constraint becomes binding; in the case of quantity constraints, an endogenous tax or subsidy is introduced.

In our algebraic exposition of equilibrium conditions below, we state the associated equilibrium variables in brackets. Furthermore, we use the notation $\Pi^{Z}_{gr}$ to denote the unit profit function (calculated as the difference between unit revenue and unit cost) for constant-returns-to-scale production of item $g$ in region $r$ where $Z$ is the name assigned to the associated production activity. Differentiating the unit profit function with respect to input and output prices provides compensated demand and supply coefficients (Hotelling’s Lemma), which appear subsequently in the market clearance conditions.

We use $g$ as an index comprising all sectors/commodities including the final consumption composite, the public good composite and an aggregate investment good. The index $r$ (aliased with $s$) denotes regions. The index $EG$ represents the subset of all energy goods except for crude oil (here: coal, refined oil, gas, electric-

\footnote{Due to non-satiation expenditure will exhaust income. Thus, the formal inequality of the income-expenditure balance will hold as an equality in equilibrium.} An example for an explicit price constraint is a lower bound on the real wage to reflect a minimum wage rate; an example for an explicit quantity constraint is the specification of a (minimum) target level for the provision of public goods.
ity) and the label X denotes the subset of fossil fuels (here: coal, crude oil, gas), whose production is subject to decreasing returns to scale given the fixed supply of fuel-specific factors. Tables 4 to 11 explain the notations for variables and parameters employed within our algebraic exposition. Figures 7 to 9 provide a graphical representation of the functional forms. Numerically, the model is implemented under GAMS (Brooke et al. 1996)\textsuperscript{24} and solved using PATH (Dirkse and Ferris 1995)\textsuperscript{25}.

Zero profit conditions

1. Production of goods except for fossil fuels ($Y_{g|r |g \not\in X}$) (see Figure 7 and Figure 10):

\[
\begin{align*}
\pi^g_{Ygr} &= \theta_{Xgr} \left( \frac{\rho_{Ygr} (1 - \tau_p - \tau_f)}{\rho_{Ygr}} \right)^{1+\eta} + (1 - \theta_{Xgr}) \left( \frac{\rho_{Ygr} (1 - \tau_p - \tau_f)}{\rho_{Ygr}} \right)^{1+\eta} \right) \frac{1}{1+\eta} \\
&- \left( \theta_{Xgr} \rho_{Ygr}^{1+\sigma_E} + (1 - \theta_{Xgr}) \left( \theta_{Xgr} \rho_{Ygr}^{1+\sigma_E} + (1 - \theta_{Xgr}) \rho_{Ygr}^{1+\sigma_E} \right) \right) \frac{1}{1+\sigma_E} \leq 0
\end{align*}
\]

2. Production of fossil fuels ($Y_{g|r |g \in X}$) (see Figure 8 and Figure 10):

\[
\begin{align*}
\pi^g_{Ygr} &= \theta_{Xgr} \left( \frac{\rho_{Ygr} (1 - \tau_p - \tau_f)}{\rho_{Ygr}} \right)^{1+\eta} + (1 - \theta_{Xgr}) \left( \frac{\rho_{Ygr} (1 - \tau_p - \tau_f)}{\rho_{Ygr}} \right)^{1+\eta} \right) \frac{1}{1+\eta} \\
&- \theta_{Xgr} \left( \rho_{Ygr}^{1+\sigma_E} + \rho_{Ygr}^{1+\sigma_E} \right) \frac{1}{1+\sigma_E} + \sum_{h \in EG} \theta_{Rhr} \left( \rho_{Athr} (1 + \tau_{Dhr} + \tau_{Dhr}^2) \right) \frac{1}{1+\sigma_D} \leq 0
\end{align*}
\]

3. Sector-specific material aggregate ($M_{gr}$):

\[
\begin{align*}
\pi^g_{Mgr} &= \rho_{Mgr} - \left( \sum_{h \in EG} \theta_{Rhr} \rho_{Athr} (1 + \tau_{Dhr} + \tau_{Dhr}^2) \right) \frac{1}{1+\sigma_D} \leq 0
\end{align*}
\]


4. Sector-specific energy aggregate ($E_{gr}$):

$$\Pi_{Egr} = PE_{gr} - \left( \Theta_{ELEgr} \left( PE_{ELEr} (1 + t_p D_{ELEgr} + t_f D_{ELEgr}) \right) \right)^{1-a_{ELE}}$$

$$\leq 0$$

5. Armington aggregate ($A_{ir}$) (see Figure 9):

$$\Pi_{A_{ir}} = PA_{ir} - \left( \Theta_{DM} \left( PE_{DMr} (1 + t_p D_{DMir} + t_f D_{DMir}) \right) \right)^{1-a_{DM}}$$

$$\leq 0$$

6. Labour supply ($L_r$):

$$\Pi_{Lr} = PL_r - \left( \Theta_{LSr} (PLS_r) \right)^{1-a_{LSr}}$$

$$\leq 0$$

7. Mobile capital supply ($K$):

$$\Pi_{K} = \left( \sum_{r} \Theta_{Kr} \left( PE_{Kr} (1 - t_p D_{Kr} - t_f D_{Kr}) \right) \right)^{1-a_{KM}}$$

$$\leq 0$$

8. Welfare ($W_r$):

$$\Pi_{Wr} = PW_r - \left( \Theta_{LSr} (PW LS) \right)^{1-a_{LSr}}$$

$$\leq 0$$

Market clearance conditions

9. Labour ($P^L_r$):

$$L_r \geq \sum_{r} \frac{\partial \Pi_{Ygr}}{\partial P_{Lr}}$$

39
10. Leisure ($P^{LS}_r$):

$$L_r - L_r \geq W_r \frac{\partial \Pi}{\partial P^{LS}_r}$$

11. Mobile capital ($P^{KM}$):

$$\sum_r K^{MR}_r \geq K$$

12. Sector-specific capital ($P^{K}_{gr}$):

$$\frac{\Pi}{\Pi^{K}_{gr}} + K \frac{\partial \Pi}{\partial P^{K}_{gr}} \geq \sum_g Y^{gr} \frac{\partial \Pi}{\partial Y^{gr}}$$

13. Fossil fuel resources ($P^{R}_{gr}$ | $g \in X$):

$$R^{gr} \geq Y^{gr} \frac{\partial \Pi}{\partial Y^{gr}} \left( \frac{1}{P^{R}_{gr}} \left( 1 + t^{P}_{Rgr} + t^{F}_{Rgr} \right) \right)$$

14. Energy composite ($P^{E}_{gr}$):

$$E^{gr} \geq Y^{gr} \frac{\partial \Pi}{\partial Y^{gr}}$$

15. Material composite ($P^{M}_{gr}$):

$$M^{gr} \geq Y^{gr} \frac{\partial \Pi}{\partial Y^{gr}}$$

16. Armington good ($P^{A}_{ir}$):

$$A^{ir} \geq \sum_g E^{gr} \frac{\partial \Pi}{\partial Y^{gr}} \left( \frac{1}{P^{E}_{gr}} \left( 1 + t^{P}_{Egr} + t^{F}_{Egr} \right) \right) + \sum_g M^{gr} \frac{\partial \Pi}{\partial Y^{gr}} \left( \frac{1}{P^{M}_{gr}} \left( 1 + t^{P}_{Mgr} + t^{F}_{Mgr} \right) \right)$$

17. Commodities ($P^{Y}_{ir}$):

$$Y^{ir} \frac{\partial \Pi}{\partial Y^{ir}} \left( \frac{1}{P^{Y}_{ir}} \left( 1 - t^{P}_{Yir} - t^{F}_{Yir} \right) \right) \geq A^{ir} \frac{\partial \Pi}{\partial Y^{ir}}$$
18. Private good consumption ($P_{Cr}^Y$):

$$Y_{Cr} \geq \frac{\partial W_r}{\partial Y_{Cr}}$$

19. Investment ($P_{Ir}^Y$):

$$Y_{Ir} \geq \bar{I}_r$$

20. Public Consumption ($P_{Gr}^Y$):

$$Y_{Gr} \geq \text{INC}_p + \theta_{Gr} \text{INC}_f$$

21. Welfare ($P_{r}^W$):

$$W_r \geq \frac{\text{INC}^{RA}}{p_{r}}$$

22. Carbon emissions ($P_{2}^{CO}$):

$$\text{CO}_2 \geq \sum_{r} \sum_{x \in g} \sum_{g \in E} q_{gr} \frac{\partial W_r}{\partial (P_{A}^{ir} (1 + \tau_p D_{igr} + \tau_f D_{igr}) + \text{INC}_r^{CO}_2)}$$

*Income-expenditure balances*

23. Income of representative consumer ($\text{INC}_r^{RA}$):

$$\text{INC}_r^{RA} = P_{LS}^r \text{L}_r + \sum_{s \in g} P_{R}^{gr} \bar{R}_{gr} + P_{KM} \text{KM}_r + \sum_{g} P_{K}^{gr} K_{gr} - P_{Y}^{Ir} \bar{I}_r + p_{CO2} \theta_{CO2} \text{CO}_2 + \mu \text{BOP}^{RA}_r - \chi \mu - \varepsilon_{r} P_{Cr}^Y$$
24. Income of provincial government \((INC^p_r)\):

\[
INC^p_r = L_r PL_r t_p L_r + \sum_{g \in x} \bar{R}_{gr} PR_{gr} t_p R_{gr} + \ldots + G_{Pr}
\]

25. Income of federal government \((INC^f)\):

\[
INC^f = \sum_r \left( (1, r^2) f^r \right) + \sum_{g} \theta_{Gr} INC^f P_Y Gr + \ldots + \bar{G}_{fr}
\]

26. Equal-yield for provincial government demand \((\chi_r)\):

\[
\frac{INC^p}{P^p_{yr}} \geq \chi_r
\]

27. Equal-yield for federal government demand \((\varepsilon)\):

\[
\sum_{r} \theta_{Gr} \frac{INC^f}{P^f_{yr}} \geq \sum v_{fr}
\]
C.1 Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>Goods excluding final demand goods</td>
</tr>
<tr>
<td>g</td>
<td>Goods including intermediate goods ((g = i)) and final demand goods, i.e. private consumption ((g = C)), investment ((g = I)) and public consumption ((g = G))</td>
</tr>
<tr>
<td>r (alias s)</td>
<td>Regions</td>
</tr>
<tr>
<td>EG</td>
<td>Energy goods: coal, refined oil, gas and electricity</td>
</tr>
<tr>
<td>X</td>
<td>Fossil fuels: coal, crude oil and gas</td>
</tr>
</tbody>
</table>

Table 4: Sets

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ygr</td>
<td>Production of good (g) in region (r)</td>
</tr>
<tr>
<td>Egr</td>
<td>Production of energy composite for good (g) in region (r)</td>
</tr>
<tr>
<td>Mgr</td>
<td>Production of material composite for good (g) in region (r)</td>
</tr>
<tr>
<td>Aigr</td>
<td>Production of Armington good (i) in region (r)</td>
</tr>
<tr>
<td>Lr</td>
<td>Labour supply in region (r)</td>
</tr>
<tr>
<td>Kr</td>
<td>Capital supply</td>
</tr>
<tr>
<td>Wr</td>
<td>Production of composite welfare good</td>
</tr>
</tbody>
</table>

Table 5: Activity variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>pYgr</td>
<td>Price of good (g) in region (r)</td>
</tr>
<tr>
<td>pEgr</td>
<td>Price of energy composite for good (g) in region (r)</td>
</tr>
<tr>
<td>pMgr</td>
<td>Price of material composite for good (g) in region (r)</td>
</tr>
<tr>
<td>pAigr</td>
<td>Price of Armington good (i) in region (r)</td>
</tr>
<tr>
<td>pLr</td>
<td>Price of labour (wage rate) in region (r)</td>
</tr>
<tr>
<td>pSLS</td>
<td>Price of leisure in region (r)</td>
</tr>
<tr>
<td>pk</td>
<td>Price of capital services (rental rate) in sector (g) and region (r)</td>
</tr>
<tr>
<td>pRg</td>
<td>Rent to fossil fuel resources in fuel production in sector (g) ((g \in X)) and region (r)</td>
</tr>
<tr>
<td>pC0gr</td>
<td>(CO_2) price in region (r)</td>
</tr>
<tr>
<td>pkm</td>
<td>Price of interregionally mobile capital</td>
</tr>
<tr>
<td>pKim</td>
<td>Price of sector-sector specific capital</td>
</tr>
<tr>
<td>pW</td>
<td>Price of composite welfare (utility) good</td>
</tr>
<tr>
<td>µ</td>
<td>Exchange rate</td>
</tr>
</tbody>
</table>

Table 6: Price variables
Table 7: Income variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>INC_{RA}^r</td>
<td>Income of representative agent in region r</td>
</tr>
<tr>
<td>INC_{p}^r</td>
<td>Income of provincial government in region r</td>
</tr>
<tr>
<td>INC_{f}</td>
<td>Income of federal government</td>
</tr>
</tbody>
</table>

Table 8: Tax rates and reference prices

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>τ_p^g</td>
<td>Provincial taxes on output in sector g and region r</td>
</tr>
<tr>
<td>τ_f^g</td>
<td>Federal taxes on output in sector g and region r</td>
</tr>
<tr>
<td>τ_p^g</td>
<td>Provincial taxes on resource extraction in sector g and region r</td>
</tr>
<tr>
<td>τ_f^g</td>
<td>Federal taxes on resource extraction in sector g and region r</td>
</tr>
<tr>
<td>τ_p^i</td>
<td>Provincial taxes on intermediate good i in sector g and region r</td>
</tr>
<tr>
<td>τ_f^i</td>
<td>Federal taxes on intermediate good i in sector g and region r</td>
</tr>
<tr>
<td>τ_p^i</td>
<td>Provincial taxes on labour in region r</td>
</tr>
<tr>
<td>τ_f^i</td>
<td>Federal taxes on labour in region r</td>
</tr>
<tr>
<td>τ_p^i</td>
<td>Provincial taxes on capital in region r</td>
</tr>
<tr>
<td>τ_f^i</td>
<td>Federal taxes on capital in region r</td>
</tr>
<tr>
<td>ð</td>
<td>Reference price of good g in region r</td>
</tr>
<tr>
<td>ðv</td>
<td>Reference value of exchange rate</td>
</tr>
<tr>
<td>ðf</td>
<td>Reference price of fossil fuel resource g in region r</td>
</tr>
<tr>
<td>ða</td>
<td>Reference price of Armington good i in region r</td>
</tr>
<tr>
<td>ðl</td>
<td>Reference price of labour (wage rate) in region r</td>
</tr>
<tr>
<td>ðk</td>
<td>Reference price of capital in region r</td>
</tr>
</tbody>
</table>

Table 9: Cost shares

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ð_{EX}^g</td>
<td>Value share of international market exports in domestic production of good g in region r</td>
</tr>
<tr>
<td>ð_{E}^g</td>
<td>Value share of energy in the production of good g in region r</td>
</tr>
<tr>
<td>ð_{M}^g</td>
<td>Value share of the material aggregate within the composite of value-added and material in the production of good g in region r</td>
</tr>
<tr>
<td>ð_{L}^g</td>
<td>Value share of labour in the value-added composite of good g production in region r</td>
</tr>
<tr>
<td>ð_{ELE}</td>
<td>Value share of electricity in the energy composite of good g production in region r</td>
</tr>
<tr>
<td>ð_{COA}^i</td>
<td>Value share of coal in the coal-oil-gas composite of good g production in region r</td>
</tr>
<tr>
<td>ð_{OIL}^i</td>
<td>Value share of oil in the oil-gas composite of good g production in region r</td>
</tr>
<tr>
<td>ð_{MM}^i</td>
<td>Value share of domestically produced inputs to Armington production of good g in region r</td>
</tr>
<tr>
<td>ð_{MM}^i</td>
<td>Value share of imports from region s in the import composite of good i to region r</td>
</tr>
<tr>
<td>ð_{K}^s</td>
<td>Value share of capital supply to region r in overall (mobile) capital supply</td>
</tr>
<tr>
<td>ð_{LS}^s</td>
<td>Value share of leisure demand in region r</td>
</tr>
<tr>
<td>ð_{CO2}^s</td>
<td>Share of region r in overall CO2 emission endowment</td>
</tr>
</tbody>
</table>

Table 9: Cost shares
Symbol | Description
--- | ---
$L_r$ | Aggregate time (labour and leisure) endowment of region $r$
$K_{gr}$ | Sector-specific capital endowment of region $r$
$R_{gr}$ | Endowment of fossil fuel resource $g$ by region $r$ ($g \in X$)
$BOP_r^R$ | Representative agent’s balance of payment deficit or surplus in region $r$
$BOP_r^P$ | Provincial government’s balance of payment deficit or surplus in region $r$
$BOP_f^P$ | Federal government’s initial balance of payment deficit or surplus
$CO_2$ | Endowment with carbon emission rights
$\gamma_{CO_2}$ | Carbon emissions coefficient for fossil fuel $i$ ($i \in X$) in good $g$ production of region $r$
$I$ | Exogenous investment demand
$G_r^p$ | Exogenous provincial government demand
$G_r^f$ | Exogenous federal government demand

Table 10: Endowments and emissions coefficients

Symbol | Description
--- | ---
$\chi_r$ | Lump sum transfers to warrant equal-yield constraint for provincial government $r$
$\varepsilon_r$ | Lump sum transfers to warrant equal-yield for federal government

Table 11: Additional variables

Figure 7: Cost function for non-fossil fuel sectors (region $(r)$ subscripts dropped to reduce notational clutter.)
Figure 8: Cost function for fossil fuel sectors (region \((r)\) subscripts dropped to reduce notational clutter.)

Figure 9: Cost function for Armington good \(i\) in region \(r\)

Figure 10: Transformation of output of good \(i\) in region \(r\)
D Decomposition

We illustrate the decomposition with a stylized version of the multi-region model, introduced in Section 5, in which we omit any details regarding functional forms and make some simplifications for ease of exposition. We summarize the equilibrium conditions in the stylized multi-region model in Table 13 and the associated notation in Table 12. Each production sector produces at constant returns to scale, and earns zero profit in equilibrium. We write a unit profit function for each sector, which is defined as a revenue function less an expenditure function. We suppress details of these functions here. There are three classes of profit functions in our stylized multi-region model: one for production of output (in each sector and region), one for production of the Armington good (for each commodity and region), and one for production of the final demand good (for each region).

Differentiating the profit function by an input or output price generates a compensated demand or supply function for the good associated with the price. These compensated demand functions are used to express the market clearance conditions associated with the equilibrium. There are four classes of market clearance condition in our stylized multi-region model: one for factor markets (for each factor and region), one for output markets (for each commodity and region), one for goods markets (for each commodity and region), and one for the consumption good market (for each region).

Finally, the model is closed by specifying income balance equations for each of the agents in the model and by fixing the balance of international payments at the benchmark level. The income of the representative agent is the sum of returns to (fixed) endowments as well as an exogenous balance of payments. The income of the federal government is due to tax revenues associated with federal taxes, which depend on tax rates and the tax base (for which details are omitted). The income of the provincial government is due to tax revenues associated with provincial taxes, including the tax on carbon emissions; again we omit details of the tax bases. In the stylized model, we include only one type of tax (in addition to the carbon tax)

\footnote{The results in the paper are produced with the complete version of the model. We use a simplified version here to reduce notational burden.}
for notational simplicity. In the numerical model, there are a variety of taxes imposed by both levels of government as described in section 4. When we conduct policy simulations by imposing an exogenous carbon tax, we impose constraints to maintain real expenditures by provincial and federal governments fixed at benchmark levels (i.e., we fix $\frac{Inc_{pg}}{p_r}$ and $\frac{Inc_{fg}}{p_r}$, respectively). We ensure real government expenditure is fixed by endogenously adjusting federal and provincial tax rates in the model ($t_f^{Y}$ and $t_p^{Y}$, respectively), or by implementing lump sum transfers, depending on the scenario.

The carbon tax is set and revenue from the carbon tax is disbursed as described in the paper. From the multi-region model, we obtain the full general equilibrium effect of the carbon tax on welfare, which we measure as a percent change in Hicksian Equivalent Variation (HEV) of income from the benchmark. To decompose the total welfare into the two terms described in Section 5, we use a single-region variant of our multi-region model.

The single-region variant of our model is described in stylized form in Table 14. It is identical to the multi-region model, with three exceptions. First, it contains only one region, and as a result regional subscripts are dropped in Table 14. Second, it treats prices facing the province ($\bar{P}_i^{Y}$) as parametric, rather than endogenous (the overbar denotes a parameter). Third, it treats federal government tax rates ($t_f^{Y}$) as parametric, rather than endogenous. Fourth, it treats the income of the federal government ($Inc^{fg}$) from other provinces as parametric, rather than endogenous.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>Number of commodities (sectors)</td>
</tr>
<tr>
<td>$K$</td>
<td>Number of factors</td>
</tr>
<tr>
<td>$M$</td>
<td>Number of regions (provinces)</td>
</tr>
<tr>
<td>$i, j = 1, \ldots, N$</td>
<td>Index for commodities (sectors)</td>
</tr>
<tr>
<td>$f = 1, \ldots, K$</td>
<td>Index for factors</td>
</tr>
<tr>
<td>$r, s = 1, \ldots, M$</td>
<td>Index for regions</td>
</tr>
<tr>
<td>$v$</td>
<td>Revenue function</td>
</tr>
<tr>
<td>$w$</td>
<td>Cost function</td>
</tr>
<tr>
<td>$\Pi_{ir}$</td>
<td>Profit function for production ($z = Y$), Armington ($z = A$), or final demand ($z = C$) for good $i$ and region $r$</td>
</tr>
<tr>
<td>$E_{fr}$</td>
<td>Endowment of factor $f$ in region $r$</td>
</tr>
<tr>
<td>$Y_{ir}$</td>
<td>Production level of good $i$ in region $r$</td>
</tr>
<tr>
<td>$A_{ir}$</td>
<td>Supply of Armington good $i$ in region $r$</td>
</tr>
<tr>
<td>$C_{r}$</td>
<td>Supply of final demand good in region $r$</td>
</tr>
<tr>
<td>$B_{r}$</td>
<td>Balance of payments in region $r$</td>
</tr>
<tr>
<td>$\text{Inc}^{c}_{r}$</td>
<td>Consumer income</td>
</tr>
<tr>
<td>$\text{Inc}^{fg}_{r}$</td>
<td>Federal government income</td>
</tr>
<tr>
<td>$\text{Inc}^{pg}_{r}$</td>
<td>Provincial government income in region $r$</td>
</tr>
<tr>
<td>$p^{Y}_{ir}$</td>
<td>Price of output of good $i$ in region $r$</td>
</tr>
<tr>
<td>$p^{F}_{fr}$</td>
<td>Price of factor $f$ in region $r$</td>
</tr>
<tr>
<td>$p^{A}_{ir}$</td>
<td>Price of Armington good $i$ in region $r$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Price of foreign exchange</td>
</tr>
<tr>
<td>$t^{pg}_{ir}$</td>
<td>Provincial government tax on sector $i$ and region $r$</td>
</tr>
<tr>
<td>$t^{fg}_{ir}$</td>
<td>Federal government tax on sector $i$ and region $r$</td>
</tr>
<tr>
<td>$\bar{t}<em>{r}^{CO</em>{2}}$</td>
<td>Carbon tax on region $r$</td>
</tr>
<tr>
<td>$\omega_{r}$</td>
<td>Federal government expenditure share in region $r$</td>
</tr>
</tbody>
</table>

Table 12: Summary of notation for the stylized models
Summary of MRM equilibrium conditions

Zero profit

Production

\[ \Pi^Y_{ir} = v(p^Y_{i1}, \ldots, p^Y_{iM}, \mu) - w(p^F_{1r}, \ldots, p^K_{ir}, p^A_{N_r}, t_fY_{ir}, t_rY_{r}, t_rY_{CO2}) \]

Armington good

\[ \Pi^A_{ir} = p^A_{ir} - w(\mu, p^Y_{i1}, \ldots, p^Y_{iM}) \]

Final demand

\[ \Pi^C_r = p^C_r - w(p^A_{1r}, \ldots, p^A_{N_r}, t_pY_{ir}, t_fY_{ir}, t_rY_{r}, t_rY_{CO2}) \]

Market clearance

Factor markets

\[ \bar{E}_{fr} = \sum_{i=1}^{N} \frac{\partial \Pi^Y_{Y_{fr}}}{\partial p^Y_{fr}} \]

Output markets

\[ Y^M_{ir} \frac{\partial \Pi^Y_{Y_{fr}}}{\partial p^M_{fr}} = \sum_{s=1}^{M} \sum_{j=1}^{N} A_{js} \frac{\partial \Pi^A_{Y_{fr}}}{\partial p^M_{fr}} \]

Goods markets

\[ A_{ir} = \sum_{j=1}^{N} Y^M_{jr} \frac{\partial \Pi^Y_{Y_{fr}}}{\partial p^A_{fr}} + C_r \frac{\partial \Pi^C_{fr}}{\partial p^A_{fr}} \]

Consumption good market

\[ p^C_r C_r = Inc^C_r + \omega_r Inc^{fg} + Inc^{pg}_r \]

Balance of payments

\[ \sum_{i=1}^{N} \sum_{r=1}^{M} Y^M_{ir} \frac{\partial \Pi^Y_{Y_{fr}}}{\partial \mu} = \sum_{i=1}^{N} \sum_{r=1}^{M} A_{ir} \frac{\partial \Pi^A_{Y_{fr}}}{\partial \mu} - \sum_{r=1}^{M} B_r \mu \]

Income balance

Consumer

\[ Inc^c_r = \bar{B}_r + \sum_{f=1}^{K} \bar{E}_{fr} p^F_{fr} \]

Federal government

\[ Inc^{fg} = v(t_fY_{ir}, \ldots) \]

Provincial government

\[ Inc^{pg}_r = v(t_pY_{ir}, t_rY_{CO2}, \ldots) \]

Table 13: Algebraic summary of the stylized multi-region (MRM) model
Summary of SRM equilibrium conditions

**Zero profit**

Production

\[ \Pi^Y_i = v(p_{i1}^Y, \ldots, p_i^Y, \ldots, p_{iM}^Y, \bar{\mu}) - w(p_1^F, \ldots, p_K^F, p_1^A, \ldots, p_N^A, t_{j1}^Y, t_{f1}^Y, t_{CO2}) \]

Armington good

\[ \Pi^A_i = p_i^A - w(\bar{\mu}, \bar{p}_{i1}^Y, \ldots, p_i^Y, \ldots, p_{iM}^Y) \]

Final demand

\[ \Pi^C = \rho^C - w(p_1^A, \ldots, p_N^A, t_{f1}^Y, t_{j1}^Y, t_{CO2}) \]

**Market clearance**

Factor markets

\[ \bar{E}_f = \sum_{i=1}^N \frac{\partial \Pi^Y_i}{\partial \bar{p}_f} \]

Output markets

\[ Y_i \frac{\partial \Pi^Y_i}{\partial p_f} = \sum_{j=1}^M \sum_{j=1}^N A_{jir} \frac{\partial \Pi^A_i}{\partial \bar{p}_f} \]

Goods markets

\[ A_{ir} = \sum_{j=1}^N Y_{jr} \frac{\partial \Pi^Y_r}{\partial \bar{p}_f} + C_r \frac{\partial \Pi^C_r}{\partial \bar{p}_f} \]

Consumption good market

\[ \rho^C \frac{\partial \Pi^C_r}{\partial \bar{p}_f} = Inc_r^C + \omega_r \bar{Inc}_{fg} + Inc_{pg}^r \]

Balance of payments

\[ \sum_{i=1}^N Y_{i} \frac{\partial \Pi^Y_i}{\partial \bar{\mu}} \bar{\mu} + \sum_{i=1}^N \sum_{r=1}^M Y_r \frac{\partial \Pi^Y_r}{\partial \bar{p}_f} \bar{p}_f = \]

\[ \sum_{i=1}^N A_{ir} \frac{\partial \Pi^A_i}{\partial \bar{\mu}} \bar{\mu} + \sum_{i=1}^N \sum_{r=1}^M A_{ir} \frac{\partial p_{ir}^Y}{\partial \bar{p}_f} \bar{p}_f - \bar{B} \bar{\mu} \]

**Income balance**

Consumer

\[ Inc^c = \bar{B} + \sum_{k=1}^K \bar{E}_{fr} p_{fr}^F \]

Federal government

\[ Inc_{fg} = v(t_{j1}^Y, \ldots) \]

Provincial government

\[ Inc_{pg} = v(t_{f1}^Y, t_{j1}^Y, t_{CO2}, \ldots) \]

Table 14: Algebraic summary of the stylized single-region (SRM) model
A condition for the generation of the type of vertical fiscal externality that we assume in this paper is that the federal government does not respond to changes in provincial tax rates in a way that eliminates the vertical fiscal externality. For example, if the federal government reduces expenditure in a province following a reduction in federal tax revenue from that province, the welfare effect we document in our analysis will not be realized. In this appendix, we use data on Canadian federal government response to changes in tax revenue by province to show that the federal government does not reduce transfers to a province or expenditures in a province following a reduction in tax receipts from that province.

Specifically, we estimate coefficients in the following equation:

$$\text{expend}_{rt} = \beta_0 + \beta_1 \text{revenue}_{rt} + \beta_2 X_{rt} + \delta_r + \gamma_t + \epsilon_{rt} \tag{1}$$

where \(\text{expend}_{rt}\) is federal government expenditure in province \(r\) in time \(t\); \(\text{revenue}_{rt}\) is federal government tax revenue from province \(r\) in time \(t\); \(X_{rt}\) is a set of demographic controls including the total population, the total population of people below 18, and the total population of people above 65; \(\delta_r\) is a province fixed effect; \(\gamma_t\) is a time fixed effect; and \(\epsilon_{rt}\) is an error term. With the inclusion of the fixed effects and controls, we can interpret the coefficient \(\beta_1\) as the change in federal government expenditures in a province associated with a one dollar change in federal government revenue from that province. A value of \(\beta_1 > 0\) implies that when federal government revenue from a province falls, expenditure in the province also falls. If \(\beta_1 > 0\), this would provide some evidence that the federal government does respond strategically to changes in provincial revenue, and this effect would work in an opposite direction to the effect in the main body of our paper. A value of \(\beta_1 < 0\) implies that when federal government revenue from a province falls, federal government expenditure in the province increases. This would amplify our core finding.

Federal government expenditures in Canada take three main forms. Transfers to persons form the bulk of federal government expenditure. Transfers to persons include the main social insurance programs operated by the federal government,
including employment insurance, old age security, and low-income and child tax credits. Transfers to provincial governments make up about one quarter of federal government outlays. Transfers to provincial governments include the Canada Health Transfer, the Canada Social Transfer, and the Equalization program. The Equalization program is formula-based and depends on a province’s fiscal capacity (tax base), with a lower tax base generating more transfers from the federal government for a province (such that $\beta_1 < 0$ in theory for this expenditure). Finally, direct federal government expenditures make up most of the rest of federal government outlays.

We use data from Statistics Canada on federal government budget balance to estimate (1); the data includes federal government expenditure by category in each province, as well as federal government tax revenue from each province.\footnote{We use data on government revenue and expenditure from Statistics Canada (2015b), along with population estimates from Statistics Canada (2015a).}

Results of the estimation are given in Table 15. We conduct separate regressions for each of the three categories of federal government expenditures. In each case $\hat{\beta}_1 < 0$, and the coefficient is highly significant. For transfers to persons, we find that a fall in federal government revenue from a province by $1$ is associated with an increase in federal government expenditure in that province by $0.11$. For federal government transfers to provinces, we find a corresponding value of $0.15$, and for federal government expenditures, we find a value of $0.08$. In total, the results suggest that a fall in federal government tax revenue from a province of $1$ is associated with an increase in federal government expenditure in that province by $0.32$ (the sum of coefficients from each individual regression). This suggests that existing mechanisms for federal government outlays are likely to amplify, rather than nullify, the effect we report in this paper.

We also supplement our OLS regressions reported in Table 15 with instrumental variables regressions. We instrument federal government revenue from provinces with provincial marginal tax rates. Because provincial taxes in Canada are progressive (and so marginal rates differ by income), we determine the marginal tax rates for four representative different levels of income in each province in each year, and
use these as instruments for federal revenue from provinces.\textsuperscript{28} The first-stage regression is identical in each case, and the F-statistic is high, suggesting relevant instruments (the first-stage F statistic has a value of roughly 300). Our instrumental variables results have identical sign and similar magnitude to the OLS results, as shown in Table 16.

\begin{table}[h]
\centering
\begin{tabular}{lccc}
\hline
 & transfer to persons & transfer to provinces & expend on goods \\
\hline
revenue & $-0.101^{***}$ & $-0.149^{***}$ & $-0.076^{***}$ \\
 & (0.014) & (0.019) & (0.015) \\
\hline
Year FE & Y & Y & Y \\
Prov FE & Y & Y & Y \\
Demography controls & Y & Y & Y \\
\hline
Observations & 290 & 290 & 290 \\
$R^2$ & 0.998 & 0.984 & 0.998 \\
\hline
\end{tabular}
\caption{Regression estimates for the model of federal government expenditures.}
\end{table}

\textit{Note:} $^*p<0.1; ^{**}p<0.05; ^{***}p<0.01$

\textsuperscript{28}Provincial marginal tax rates are estimates from Milligan (2012).
<table>
<thead>
<tr>
<th></th>
<th>transfer to persons</th>
<th>transfer to provinces</th>
<th>expend on goods</th>
</tr>
</thead>
<tbody>
<tr>
<td>revenue</td>
<td>−0.060</td>
<td>−0.116**</td>
<td>−0.190***</td>
</tr>
<tr>
<td></td>
<td>(0.038)</td>
<td>(0.052)</td>
<td>(0.044)</td>
</tr>
<tr>
<td>Year FE</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Prov FE</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Demography controls</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Observations</td>
<td>250</td>
<td>250</td>
<td>250</td>
</tr>
</tbody>
</table>

Note: *p<0.1; **p<0.05; ***p<0.01

Table 16: Instrumental variables estimates for the model of federal government expenditures.
Working Papers of the Center of Economic Research at ETH Zurich

(PDF-files of the Working Papers can be downloaded at www.cer.ethz.ch/research).

16/234 C. Boehringer, N. Rivers, H. Yonezawa
   Vertical fiscal externalities and the environment

16/233 J. Abrell and S. Rausch
   Combining Price and Quantity Controls under Partitioned Environmental Regulation

16/232 L. Bretschger and A. Vinogradova
   Preservation of Agricultural Soils with Endogenous Stochastic Degradation

16/231 F. Lechthaler and A. Vinogradova
   The Climate Challenge for Agriculture and the Value of Climate Services: Application to Coffee-Farming in Peru

16/230 S. Rausch and G. Schwarz
   Household heterogeneity, aggregation, and the distributional impacts of environmental taxes

16/229 J. Abrell and S. Rausch
   Cross-Country Electricity Trade, Renewable Energy and European Transmission Infrastructure Policy

16/228 M. Filippini, B. Hirl, and G. Masiero
   Rational habits in residential electricity demand

16/227 J. Abrell, S. Rausch, and H. Schwerin
   Long-Run Energy Use and the Efficiency Paradox

15/226 L. Bretschger, F. Lechthaler, S. Rausch, and L. Zhang
   Knowledge Diffusion, Endogenous Growth, and the Costs of Global Climate Policy

15/225 H. Gersbach
   History-bound Reelections

15/224 J.-P. Nicolai
   Emission Reduction and Profit-Neutral Permit Allocations

15/223 M. Miller and A. Alberini
   Sensitivity of price elasticity of demand to aggregation, unobserved heterogeneity, price trends, and price endogeneity: Evidence from U.S. Data

15/222 H. Gersbach, P. Muller and O. Tejada
   Costs of Change, Political Polarization, and Re-election Hurdles
15/221 K. Huesmann and W. Mimra
Quality provision and reporting when health care services are multi-dimensional and quality signals imperfect

15/220 A. Alberini and M. Filippini
Transient and Persistent Energy Efficiency in the US Residential Sector: Evidence from Household-level Data

15/219 F. Noack, M.-C. Riekhof, and M. Quaas
Use Rights for Common Pool Resources and Economic Development

15/218 A. Vinogradova
Illegal Immigration, Deportation Policy, and the Optimal Timing of Return

15/217 L. Bretschger and A. Vinogradova
Equitable and effective climate policy: Integrating less developed countries into a global climate agreement

15/216 M. Filippini and L. C. Hunt
Measurement of Energy Efficiency Based on Economic Foundations

15/215 M. Alvarez-Mozos, R. van den Brink, G. van der Laan and O. Tejada
From Hierarchies to Levels: New Solutions for Games with Hierarchical Structure

15/214 H. Gersbach
Assessment Voting

15/213 V. Larocca
Financial Intermediation and Deposit Contracts: A Strategic View

15/212 H. Gersbach and H. Haller
Formal and Real Power in General Equilibrium

15/211 L. Bretschger and J. C. Mollet
Prices vs. equity in international climate policy: A broad perspective

15/210 M. Filippini and F. Heimsch
The regional impact of a CO2 tax on gasoline demand: a spatial econometric approach

15/209 H. Gersbach and K. Wickramage
Balanced Voting

15/208 A. Alberini and C. Towe
Information v. Energy Efficiency Incentives: Evidence from Residential Electricity Consumption in Maryland

14/207 A. Bommier
A Dual Approach to Ambiguity Aversion
14/206 H. Gersbach, U. Schetter and M. T. Schneider
Taxation, Innovation, and Entrepreneurship

14/205 A. Alberini and A. Bigano
How Effective Are Energy-Efficiency Incentive Programs? Evidence from Italian Homeowners

14/204 D. Harenberg and A. Ludwig
Social Security in an Analytically Tractable Overlapping Generations Model with Aggregate and Idiosyncratic Risk

14/203 A. Bommier, L. Bretschger and F. Le Grand
Existence of Equilibria in Exhaustible Resource Markets with Economies of Scale and Inventories

14/202 L. Bretschger and A. Vinogradova
Growth and Mitigation Policies with Uncertain Climate Damage

14/201 L. Bretschger and L. Zhang
Carbon policy in a high-growth economy: The case of China

14/200 N. Boogen, S. Datta and M. Filippini
Going beyond tradition: Estimating residential electricity demand using an appliance index and energy services

14/199 V. Britz and H. Gersbach
Experimentation in Democratic Mechanisms

14/198 M. Filippini and E. Tosetti
Stochastic Frontier Models for Long Panel Data Sets: Measurement of the Underlying Energy Efficiency for the OECD Countries

14/197 M. Filippini and W. Greene
Persistent and Transient Productive Inefficiency: A Maximum Simulated Likelihood Approach

14/196 V. Britz, P. J.-J. Herings and A. Predtetchinski
Equilibrium Delay and Non-existence of Equilibrium in Unanimity Bargaining Games

14/195 H. Gersbach, M. T. Schneider and O. Tejada
Coalition-Preclusion Contracts and Moderate Policies

14/194 A. Bommier
Mortality Decline, Impatience and Aggregate Wealth Accumulation with Risk-Sensitive Preferences

14/193 D. Harenberg and A. Ludwig
Social Security and the Interactions Between Aggregate and Idiosyncratic Risk